ADAPTIVE DISCRETE GENERALIZED LIFTING FOR LOSSLESS COMPRESSION

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ABSTRACT

A method for lossless coding using a multi-resolution scheme with an adaptive lifting step is introduced in this paper. To this goal, a generalized lifting scheme, where the sums included in the classical lifting are generalized to include possibly non linear and/or adaptive operations, is proposed. The conditions for the reversibility of the scheme are analyzed. Although the framework is valid for any type of sampled signals, we focus in particular on the discrete case (signals represented with a finite number of bits). Finally, experiments with a generalized prediction step are reported. They show the interest of the approach for lossless compression.

1. INTRODUCTION

In this paper, a wavelet based multi-resolution decomposition for image compression is described. The motivation of this work comes from the fact that, in current schemes, textures and edges need the major part of the bit-rate because wavelets are optimal bases for many signal classes with some smoothness. However, they are much less efficient representing singularities. In fact, a compromise arises when choosing the wavelet: some are more adequate for smooth regions and others behave better near discontinuities. Hence, many researchers have proposed adaptive schemes that modify the underlying wavelet bases according to the local signal characteristics. Initially, the complexity and challenge was to assure the reversibility of the filter banks. Several works attained this goal. Later, the lifting scheme by Sweldens [8] gave a suitable framework for developing time-varying wavelet filter banks thanks to the initial polyphase decomposition (or Lazy Wavelet Transform, LWT) which allows all kind of signal statistical analysis of one branch in order to apply a good filter on the other one, and furthermore, this analysis is reproducible at reception (synthesis) and thus the transform is reversible. Many contributions follow this idea, like [1, 2, 3, 5, 9] or the one by Egger et al. [4] which can be understood within the lifting framework. These works try in many different ways to exploit the correlation existing among both branches of the decomposition and considering the local form or statistics of one signal to obtain a good prediction / interpolation of the other one.

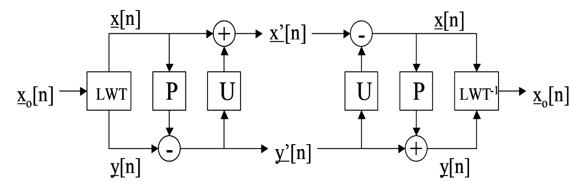

Fig.1. Lifting Scheme

Going one step further, adaptation may be improved if the information given by the same branch to be filtered is considered. There exist methods for searching the best basis for the entire signal or for a class of signals, but this requires book-keeping and so additional bit-rate to attain the reversibility. A point-wise adaptation is also possible by using a criterion invariant to the filtering so that it can be recovered at the decoder and allow choosing the correct synthesis filter [6].

Following this line of research, in this paper, we propose a generalized lifting scheme where the classical sums included in the lifting scheme are generalized to include possibly non linear and/or adaptive operations. Furthermore, we propose a general condition to guarantee the reversibility of the generalized scheme. Finally, we propose a specific generalized lifting scheme and illustrate its interest for lossless compression.

In Section 2, the classical and the adaptive lifting are briefly reviewed. The generalized lifting scheme is discussed in section 3. Section 4 focuses on the discrete case and section 5 presents a specific generalized prediction. Experiments are reported in section 6 and finally, conclusions are established in section 7.

2. LIFTING AND ADAPTIVE LIFTING

The lifting scheme (Fig. 1) introduced by Sweldens is a well-known method to create bi-orthogonal wavelet filters from other ones. Usually, a polyphase decomposition (LWT) of the input signal $\underline{x}_0$ is done, obtaining an approximation signal $\underline{x}$ and a detail signal $\underline{y}$. Then, lifting steps are performed by predicting the detail signal from the $\underline{x}$ samples (1) and updating the approximation signal with the $\underline{y}$ samples (2). The so-called prediction and update steps improve the initial lazy wavelet properties

$$y'[n] = y[n] - P[\underline{x}] \quad (1)$$

$$x'[n] = x[n] + U[\underline{y}'] \quad (2)$$

Several steps (prediction + update or vice versa) may be concatenated in order to reach the desired properties for the wavelet basis. These prediction and update operators may be a linear combination of $\underline{x}$ and $\underline{y}$, respectively, or any non-linear operation, since by construction the lifting scheme is always reversible.

The adaptive lifting scheme [2, 5, 6] is a modification of the classical lifting. Figure 2 illustrates an example of adaptive update step followed by a fixed prediction step. At each sample n, an update operator is chosen according to a decision function D which depends on $\underline{y}$ as in [2, 5], but may also depend on the sample $x[n] \in \underline{x}$ being updated [6]. In this last case, a problem arises because the decoder does not know the sample $x[n]$ used at the coder for the decision. Instead, the decoder has access to $x'[n]$, the updated version of $x[n]$ through an unknown update filter. Therefore, the challenge is to find a decision function and a set of filters which allow us to reproduce the decision $D(x[n],\underline{y})$ at the decoder (3), thus obtaining a reversible decomposition scheme. This property is known as the "decision conservation condition" [6]:

$$D(x[n],\underline{y}) = D'(x'[n],\underline{y}) \qquad (3)$$

In practice, the range of D may indicate that there exists an edge on $x[n]$ if D is the L1-norm of the gradient, or that $x[n]$ corresponds to a textured region if a texture detector D is used (like in [4]), or may indicate other geometrical constraints. Depending on the local characteristics of the signal at n assessed by D, a suitable filter for these characteristics may be chosen. A classical strategy consists of using low-pass filters for smooth regions and short-support filters for edges.

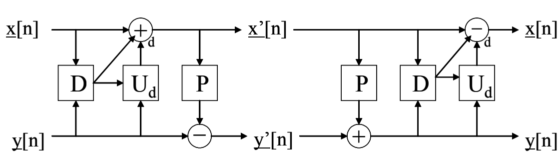

Fig. 2. Adaptive Update Lifting Scheme

3. ADAPTIVE GENERALIZED LIFTING

In this section, we propose the generalization of the lifting scheme illustrated in Fig. 3. As can be seen, the scheme is very similar to the classical lifting except that the sums after the prediction and the update are embedded in a more general framework. The prediction is viewed here as a function that maps $y[n]$ to $y'[n]$ taking into account values from $\underline{x}$. In a classical lifting, the prediction is viewed as a filter that generates a value that is used to modify $y[n]$ through a sum. In the generalized scheme, we have removed the restriction of modifying $y[n]$ only through a sum and open the door to more complex, possibly adaptive or nonlinear modifications. A similar generalization is done for the update.

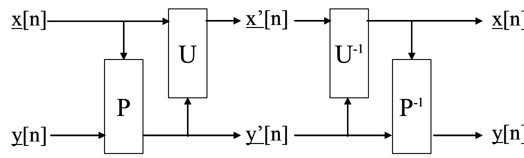

Fig. 3. Generalized Lifting Scheme

Of course, in order to have a reversible scheme, the generalized prediction and update cannot be chosen arbitrarily. Let us analyze the restriction we have to impose, for example, on the generalized update:

Let A be the space of functions a from $\Re \times \Re^k$ to itself:
$a \in A \Leftrightarrow a : \Re \times \Re^k \rightarrow \Re \times \Re^k$, such that:
$$\{z_1'[n], z_2'[n-n_1], \ldots, z_2'[n-n_k]\} = a\,\{z_1[n], z_2[n-n_1], \ldots, z_2[n-n_k]\} \qquad (4)$$

Let A_0 be the subspace of A containing all functions that do not modify $z_2[n]$, that is for which the restriction to $\Re^k$ is the identity: $A_o = \{a \subseteq A \mid a_{|\Re^k \rightarrow \Re^k} = Id\}$.

In the sequel, we consider a generalized update as a function of A to highlight the dependency with respect to the samples. But, as it can only modify $x[n]$, it should be a function of A_0. In order to have a reversible scheme, the generalized update should be a bijective function of A_0. The same analysis can be done in the case of the generalized prediction. As a result, both the generalized prediction and update should be bijective functions of A_0.

4. DISCRETE CASE

The scheme presented in section 3 assumes that the values taken by $\underline{x}$ and $\underline{y}$ are real numbers. In many applications related to compression, the values of $x[n]$ and $y[n]$ are quantized before transmission. In this case, it is the mapping $Q(a\{z_1[n], z_2[n-n_1], \ldots, z_2[n-n_k]\}$, where Q represents the quantization, that should be a bijective function of A_0. We have found several reversible schemes that include quantization. However, the resulting decompositions were not suitable for compression.

An alternative approach is to consider the discrete version of the generalized lifting scheme. To this goal, we assume that the values taken by $\underline{x}$ and $\underline{y}$ are integers, and that the generalized prediction and update output are also integers. In this case, no quantization is necessary after the update or the prediction and the only issue is to design a discrete bijective mapping.

Consider now the following framework for discrete gray-scale images where each pixel is represented by 8 bits. Without loss of generality, we will assume that sample values may range from -128 to 127. Let us call Z_{255} the set of integers that belong to the interval $[-128,127]$. The discrete generalized update and prediction are now functions from the $Z_{255} x Z_{255}{}^k$ space to itself that can only modify the first components. The statements made in section 3 are also valid for the discrete case. In particular, reversibility is obtained if the mappings: $\{z_1'[n], z_2[n-n_1], \ldots, z_2[n-n_k]\} = a\{z_1[n], z_2[n-n_1], \ldots, z_2[n-n_k]\}$ are bijective.

For $z_2[n-n_1], \ldots, z_2[n-n_k]$ fixed, the set of all possible values of $z_1[n]$ describes a column in the $Z_{255}xZ_{255}{}^k$ space. Let $C_{i\in Z_{255}{}^K}$ denote such a column:

$$C_{i\in Z_{255}{}^K} = \{z_1[n], z_2[n-n_1]=i_1, \ldots, z_2[n-n_k]=i_K\} \quad (5)$$

As, the generalized update and prediction can only modify the first component $z_1[n]$, they map the column $C_{i\in Z_{255}{}^K}$ to itself. In order to have a reversible scheme, the mapping of $C_{i\in Z_{255}{}^K}$ to itself should be bijective for all columns. Fig. 4 illustrates the case where k=2. To simplify the notation the column $C_{i\in Z_{255}{}^K}$ is denoted by $C_{i,j}$.

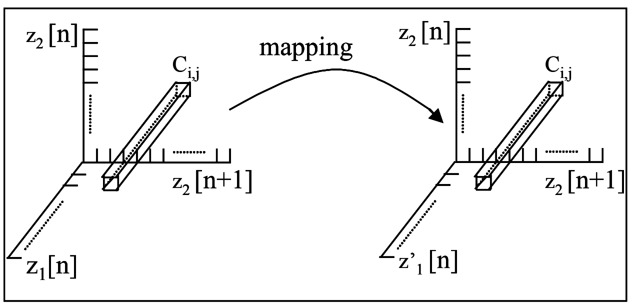

Fig. 4. Discrete mapping from $Z_{255}xZ_{255}{}^2$ to itself. The lifting step is reversible if all mappings from every column $C_{i,j}$ to itself are bijective.

The number of bijective mappings of a column to itself is equal to the factorial of 256. The mapping choice depends on two factors. First, mappings should be defined and computed easily, since arbitrary mappings require of a look-up table which size is proportional to the number of points of the discrete space. Furthermore, a great amount of operations to compute the transform is not desired.

The second factor is related to the goal of compression. Entropy coding methods like EBCOT, SPIHT [7] or the Embedded Zero-Tree coders rely on two facts to reach high-compression rates: first, in general, detail samples have small values and, second, large values are spatially related. If a discrete generalized lifting is used in combination with a classical entropy coder, it should produce wavelet-type coefficients. For instance, a generalized prediction should map the more probable points of $Z_{255}xZ_{255}{}^k$ to the smallest $z_1'[n]=y'[n]$ values.

5. ADAPTIVE DISCRETE PREDICTION

The two factors discussed above are considered now to create the mappings for a discrete generalized prediction step. For simplicity, we restrict ourselves to $k=2$, as the classical lifting with the LeGall filter, used in the JPEG2000 standard for lossless compression. In the case of 1D signal, the generalized prediction produces a detail sample $y'[n]$ from $y[n]$ and its two neighbors $x[n]$ and $x[n+1]$. This can be written as:

$$\{y'[n], x[n], x[n+1]\} = Pred\ \{y[n], x[n], x[n+1]\} \quad (6)$$

In the $Z_{255}xZ_{255}{}^2$ space, the line l: $x[n]=x[n+1]=y[n]$ plays a special role because every point p on l must be mapped to the point (0, $x[n]$, $x[n+1]$) to have a zero detail sample if the input signal is a constant. Then, the mapping of a point p:($y[n]$, $x[n]$,$x[n+1]$) is based on its relative position with respect to the line l. The distance of a point p to l is given by:

$$\begin{aligned} dist(p,l) = {} & y[n]^2 - (x[n]+x[n+1])y[n] \\ & + (x[n]^2 + x[n+1]^2 - x[n]x[n+1]) \end{aligned} \quad (7)$$

The prediction is constructed by reordering the points of a column $C_{i,j}$ according to their distance to l. The distance is a parabola with respect to $y[n]$ (eq. 7). Its minimum is reached when $y[n] = y_{min} = (x[n] + x[n+1])/2$. It is always in the range of the space. This minimum point y_{min} is mapped to zero to vanish the first moment. Then, inside the so-called "linear zone" (Fig. 5), the values below y_{min} are mapped to negative integers maintaining their order, and similarly, the values over y_{min} are mapped to positive integers. Beyond the linear zone, values are alternatively mapped to the positive and negative remaining integers. As can be seen, the mapping in the linear zone is equivalent to:

$$y'[n] = y[n] - (x[n] + x[n+1])/2 \quad (8)$$

This generalized prediction is equivalent to the classical LeGall's prediction step. Outside the linear zone, the mapping does not correspond to a simple linear filter.

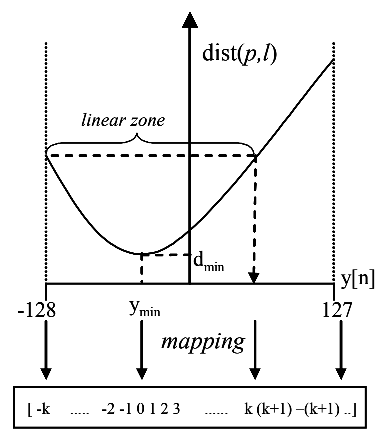

Fig. 5. Distance between the points of a column to the line l and the proposed mapping for the generalized prediction.

This mapping offers several advantages: First, it can easily be computed through a distance function. Second, if $y[n]$ falls within the linear zone, the resulting detail sample has the typical high-pass meaning. This fact is important if an update filter follows the prediction. An update is needed for multi-resolution decomposition, since posterior processing of the approximation signals performs better when it is a sampled low-pass version of original signal.

6. EXPERIMENT AND RESULTS

The generalized prediction step performances are assessed in a multiresolution framework. To this end, the scheme is completed with an adaptive update (Equ. 9), which is space-varying according to the values of the detail signal. A sample $x[n]$ is updated with two detail samples $y'[n-1]$ and $y'[n]$. If the modulus of these detail samples are small, then, as first approximation, they are high-pass samples and can be directly used for an update step as classical updates do. Detail samples with large values means, also as first approximation, that $y[n]$ comes from an edge. If smooth $\underline{x}'$ is desired, the edge should not flow to lower resolution levels and consequently, no update is performed. Values have small or large value according to a threshold T, fixed in our experiment to 12.

$$x'[n]=\begin{cases} x[n] & if \quad \max(|y'[n-1]|,|y'[n]|)>T \\ x[n]+round((y'[n-1]+y'[n])/4) & otherwise \end{cases} \tag{9}$$

Since the values in the following resolution levels may not have the same dynamic range, the generalized prediction is modified to handle an arbitrary range of values. The algorithm is the same, but the range of values has to be sent to the decoder to recover the original data.

One resolution level is obtained first by filtering every row and then only the columns of the approximation image. This leads to a three-band decomposition. The method is applied to 7 images and compared to two non-adaptive wavelet filters: the Haar and the LeGall wavelet. The decompositions are followed by the SPIHT coder [7]. The resulting bit-rates for two resolution levels are shown in Table 1. For the tested images, the proposed discrete generalized lifting scheme performs around 4.5% better than the LeGall wavelet. For three decomposition levels, results are only slightly better than LeGall's (Table 2). This decrease of gain may be due to the update filter which may not be the best choice for obtaining a good approximate signal for further processing.

Image / Filter	LeGall	Haar	Bij. Map.
Lenna	203402	197726	190924
Cameraman	55100	53064	51889
Goldhill	52731	57830	52961
Baboon	235983	243073	222286
Barbara	212913	223754	216340
Peppers	212171	217830	198506
Girl	195707	204320	182367
Mean	**166858**	**171085**	**159325**

Table 1. Bytes for lossless coding of each image using LeGall, Haar and our filter for 2 resolution levels.

Image / Filter	LeGall	Haar	Bij. Map.
Lenna	159958	171802	158304
Cameraman	42524	43604	41847
Goldhill	48545	50727	47948
Baboon	210328	220855	211209
Barbara	178565	189619	179588
Peppers	172529	180646	172876
Girl	149062	160443	145840
Mean	**137359**	**145385**	**136802**

Table 2. Results for 3 resolution levels.

7. CONCLUSIONS AND FUTURE WORK

A framework for adaptive generalized lifting has been proposed. We have focused in particular on the discrete case. Initial results for lossless image compression are promising. Adding a final update step should lead to improve performance for any number of resolution levels. Our current work centers on defining a suitable generalized update. Moreover, we are studying how to increase the support of the generalized update and prediction as well as defining 2D schemes.

7. REFERENCES

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