Processing Radar Images with Hierarchical Region-Based Representations and Graph Signal Processing Tools

Philippe Salembier

Signal Theory and Communications Department
Technical University of Catalonia
Barcelona, Spain

Samuel Foucher
Vision and Imaging Team of the Computer Research Institute of Montreal
Montreal, Canada

Sergi Liesegang
Signal Theory and Communications Department
Technical University of Catalonia
Barcelona, Spain

Carlos López-Martínez
Remote Sensing and Natural Resources
Luxembourg Institute of Science and Technology, Luxembourg
Outline

• SAR and Polarimetric SAR images

• Tree representation of images
  • Maxtree/mintree
  • Binary Partition Tree

• Tree pruning strategies
  • Pruning techniques for PolSAR images
  • Application to segmentation and speckle filtering

• Tree representation and graph signal processing
  • Filtering strategies
  • Application to ship detection on SAR images.

• Summary and Conclusions
Radar remote sensing:

- Complementary information to optical systems
- Penetration of radar waves (high penetration for long wavelengths, allows volume modeling)
- Weather independent & day-and-night imaging capability
SAR (Synthetic aperture radar) images:

• Electromagnetic waves are sequentially transmitted and backscattered echoes are recorded while the platform is moving.

• The coherent signal combination allows the construction of a virtual aperture much longer than the physical antenna length.

• Measure the scene reflectivity and SAR images are commonly displayed in terms of intensity values.

• Images are corrupted by speckle noise (presence of many elemental scatterers with a random distribution within a resolution cell. The coherent sum results in strong fluctuations of the backscattering)
Polarimetric SAR images

- Add diversity
- Acquisition of polarization states of an electromagnetic field (sensitive to the target geometry and the dielectric properties of targets)
Polarimetric SAR images

- **PolSAR images:**
  - The measured information is the scattering matrix:

  \[ S_c = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \xrightarrow{\text{Vectoriz.}} \begin{bmatrix} S_{hh} & \sqrt{2}S_{hv} & S_{vv} \end{bmatrix} \]

- Each PolSAR pixel is represented by a covariance matrix (under the Gaussian scattering assumption):

  \[ C = \begin{bmatrix} E\left\{ \left| S_{hh} \right|^2 \right\} & \sqrt{2}E\left\{ S_{hh}S_{hv}^* \right\} & E\left\{ S_{hh}S_{vv}^* \right\} \\ \sqrt{2}E\left\{ S_{hv}S_{hh}^* \right\} & 2E\left\{ \left| S_{hv} \right|^2 \right\} & \sqrt{2}E\left\{ S_{hv}S_{vv}^* \right\} \\ E\left\{ S_{vv}S_{hh}^* \right\} & \sqrt{2}E\left\{ S_{vv}S_{hv}^* \right\} & E\left\{ \left| S_{vv} \right|^2 \right\} \end{bmatrix} \]
Images are classically seen as a *set of pixels*, but pixels:
- are **very numerous**,
- carry a **small amount of information**,  
- are **unstructured**,  
- are heavily **corrupted by speckle noise** in the (Pol)SAR case.
Image representation

We would like an image representation....
• Made of more meaningful primitives,
• Involving a reduced number of primitives.
=> **Need of pixel aggregation**

The representation should support many different applications....
But, the relevant scale for the application is unknown.
=> **Need of multiscale representation**

Easy access to the information
=> **Need of structured representation**
Outline

• SAR and Polarimetric SAR images

• Tree representation of images
  • Maxtree/mintree
  • Binary Partition Tree

• Tree pruning strategies
  • Pruning techniques for PolSAR images
  • Application to segmentation and speckle filtering

• Tree representation and graph signal processing
  • Filtering strategies
  • Application to ship detection on SAR images.

• Summary and Conclusions
Maxtree/Mintree

**Maxtree**: Structuring of the binary connected components of upper level sets

Image region of support

Essentially describes maxima
Maxtree/Mintree

**Mintree**: Dual representation (lower level sets), describe minima

What about multidimensional images?
Binary Partition Tree

Iterative region merging algorithm
- Start with an initial partition (pixels or superpixel partition)
- Iteratively merge the most similar pair of neighboring regions

Essentially as a hierarchical segmentation but keep track of the merging sequence in a tree structure.
**Binary Partition Tree**

**BPT**: Hierarchical Region-based representation where

- **Leaves** represent the image pixels or regions of an initial ( oversegmented ) partition
- The **remaining nodes** represent the union of child nodes
- The **links** represent the inclusion relationship
Outline

• SAR and Polarimetric SAR images
• Tree representation of images
  • Maxtree/mintree
  • Binary Partition Tree
• Tree pruning strategies
  • Pruning techniques for PolSAR images
  • Application to segmentation and speckle filtering
• Tree representation and graph signal processing
  • Filtering strategies
  • Application to ship detection on SAR images.
• Summary and Conclusions
Image Processing with BPT

Image → Tree Creation → Search Graph cut

Initial partition → Binary partition tree
BPT construction for PolSAR images

Initial partition for PolSAR images

• It is necessary to pre-filter the data to deal with the speckle noise
• The use of super-pixel partitions provides a drastic reduction of the complexity without significant loss in quality

Iterative region merging algorithm

• Start with the super-pixel partition
• Iteratively merge the most similar pair of neighboring regions

Definition of the merging algorithm:
- How to model regions:
  \[ M(R) \quad M(R_1 \cup R_2) \]
- What is the similarity between two neighboring regions:
  \[ S(R_1, R_2) \]
**BPT construction for PolSAR images**

- **Definition of the merging algorithm:**
  - How to model regions: 
    
    \[ M(R) \quad M(R_1 \cup R_2) \]
  - What is the similarity between two neighboring regions:
    
    \[ S(R_1, R_2) \]

- **Region Model:**
  - In homogeneous areas, spatial averaging is the MLE under the Gaussian assumption. The region model is therefore defined as the region mean:
    
    \[
    M(R) = Z_R = \begin{bmatrix}
    \{ |S_{hh}|^2 \} & \sqrt{2} \{ S_{hh} S_{hv}^* \} & \{ S_{hh} S_{vv}^* \} \\
    \sqrt{2} \{ S_{hv} S_{hh}^* \} & 2 \{ |S_{hv}|^2 \} & \sqrt{2} \{ S_{hv} S_{vv}^* \} \\
    \{ S_{vv} S_{hh}^* \} & \sqrt{2} \{ S_{vv} S_{hv}^* \} & \{ |S_{vv}|^2 \}
    \end{bmatrix}
    \]
BPT construction for PolSAR images

• Region Similarity:
  • If pixels were carrying color information in the Lab space, we could use the Euclidean distance as similarity, but....
  • Similarity between covariance matrices: Geodesic distance in the manifold of hermitian positive definite matrices:

\[
S(R_1, R_2) = \| \log(Z_{R_1}^{-1/2} Z_{R_2} Z_{R_1}^{-1/2}) \|. \ln(2n_1n_2 / (n_1 + n_2))
\]

Geodesic distance

Encourage the merging of small regions

• Extracting a partition from a BPT
  • Simplest approach: Stop the merging at a given point (merging sequence truncation).
  • Partitions can also be extracted by a particular graph cut: pruning.
BPT Processing

• **Extracting a partition from a BPT**
  • Simplest approach: Stop the merging at a given point (merging sequence truncation).
  • Partitions can also be extracted by a particular graph cut: **pruning**.
  • **Pruning**: Graph cut separating the connected component including the source from the one including the sink such that **siblings falls in the same connected component**.
• **Extracting a partition from a BPT**
  - Simplest approach: Stop the merging at a given point (merging sequence truncation).
  - Partitions can also be extracted by a particular graph cut: **pruning**.
  - **Pruning**: Graph cut separating the connected component including the source from the one including the sink such that *siblings falls in the same connected component.*
BPT Processing

• Extracting a partition from a BPT
  • Simplest approach: Stop the merging at a given point (merging sequence truncation).
  • Partitions can also be extracted by a particular graph cut: **pruning**.

• **Pruning**: Graph cut separating the connected component including the source from the one including the sink such that **siblings falls in the same connected component**.
BPT Processing

• Extracting a partition from a BPT
  • Simplest approach: Stop the merging at a given point (merging sequence truncation).
  • Partitions can also be extracted by a particular graph cut: pruning.
  • Pruning: Graph cut separating the connected component including the source from the one including the sink such that siblings falls in the same connected component.
BPT Processing

- **Extracting a partition from a BPT**
  - Simplest approach: Stop the merging at a given point (merging sequence truncation).
  - Partitions can be also extracted by a particular graph cut: **pruning**.
  - **Pruning**: Graph cut separating the connected component including the source from the one including the sink such that siblings falls in the same connected component.

Not seen during the merging sequence
Optimum pruning technique

• The optimization of an h-increasing criterion (in particular additive) on the tree can be efficiently done through dynamic programming.

\[ C = \sum_R \phi_R \]

Local decisions are globally optimum

\[
\text{if } \phi_{R_0} < \phi_{R_1} + \phi_{R_2} \\
\quad \text{Select } R_0 \\
\text{else} \\
\quad \text{Select } R_1 \& R_2
\]

1) Squared Error

- Define a homogeneity criterion: 
  
  \[ C = \sum_{R} \phi_{R} \quad \text{with} \quad \phi_{R} = \sum_{i,j \in R} \left\| Z^I_{i,j} - Z_{R} \right\| \]
Pruning criteria

1) Squared Error

- Define a homogeneity criterion:
  \[ C = \sum_R \phi_R \quad \text{with} \quad \phi_R = \sum_{i,j \in R} \| Z^I_{i,j} - Z_R \| \]

- **Issue**: The criterion has to be regularized:
  \[ \phi^SE_R = \sum_{i,j \in R} \| Z^I_{i,j} - Z_R \| + \lambda \]
Pruning criteria

1) Squared Error

- Define a homogeneity criterion:
  \[ C = \sum_{R} \phi_R \text{ with } \phi_R = \sum_{i,j \in R} \| Z^I_{i,j} - Z_R \| \]

- **Issue:** The criterion has to be regularized:
  \[ \phi^{SE}_R = \sum_{i,j \in R} \| Z^I_{i,j} - Z_R \| + \lambda \]

2) SAR Squared Error

- Deal with the speckle noise (multiplicative):
  \[ \phi^{\text{SAR,SE}}_R = \sum_{i,j \in R} \frac{\| Z^I_{i,j} - Z_R \|}{\| Z_R \|} + \lambda \]
Pruning criteria

3) Wishart distance

- Similarity between covariance matrices of pixels and regions:
  \[ tr\left(\left(\mathbf{Z}_{ij}^I\right)^{-1}\mathbf{Z}_R\right) + tr\left(\mathbf{Z}_R^{-1}\mathbf{Z}_{ij}\right) \]

- **Issue**: Matrix inversion for each pixel => use only diagonal elements:

\[
\phi_R^{\text{Wishart}} = \sum_{i,j \in R} \sqrt{\sum_{k=1,2,3} \frac{Z_{ij}^I(k,k)^2 + Z_R(k,k)^2}{Z_{ij}^I(k,k)Z_R(k,k)}} + \lambda
\]
Pruning criteria

3) Wishart distance

- Similarity between covariance matrices of pixels and regions:

\[ tr((Z_{ij}^I)^{-1}Z_R) + tr(Z_R^{-1}Z_{ij}) \]

- **Issue:** Matrix inversion for each pixel => use only diagonal elements:

\[
\phi_{\text{Wishart}}^R = \sum_{i,j\in R} \sqrt{\sum_{k=1,2,3} \frac{Z_{ij}^I(k,k)^2 + Z_R(k,k)^2}{Z_{ij}^I(k,k)Z_R(k,k)}} + \lambda
\]

4) Geodesic distance

- Distance adapted to the cone of hermitian positive definite matrices:

\[
\| \log(Z_R^{-1/2}Z_{ij}^I Z_R^{-1/2}) \|
\]

- Complex => use only diagonal elements:

\[
\phi_{\text{Geodesic}}^R = \sum_{i,j\in R} \sqrt{\sum_{k=1,2,3} \ln^2 \left( \frac{Z_{ij}^I(k,k)}{Z_R(k,k)} \right)} + \lambda
\]
BPT pruning evaluation

• Use of a dataset with ground-truth:
  • Simulated PolSAR Images
  • Typical polarimetric responses have been extracted from an AIRSAR image (L-band)
  • Class regions are spatially modeled by a Markov Random Field
  • Single look images have been generated using Cholesky decomposition

Results

• Assess the quality of the extracted partitions by the **Precision and Recall curves** classically used in computer vision:
  • Classify all pairs of pixels as **boundary** or **non-boundary**
  • Compare the performances of the extracted partition compared to the ground truth


• Complete view of the system behavior (from under- to over-segmentation)
• Ideal system should have Precision and Recall close to 1
Results: Precision & Recall curves

Over-segmentation (low $\lambda$)

Best tradeoff

Under-segmentation (high $\lambda$)

Best pruning approach: SAR_SE

$$\phi_{R}^{\text{SAR,SE}} = \sum_{i,j \in R} \frac{||Z^{i}_{r} - Z_{R}||}{||Z_{R}||} + \lambda$$
Results: Precision & Recall curves

Sensitivity of the results with respect to the regularization parameter:

Summary of Precision and Recall with harmonic mean:

\[ F = \frac{2PR}{P + R} \]
Results: Precision & Recall curves

Merging sequence truncation versus Mincut

Justify the BPT-based strategy
Results: Synthetic images

<table>
<thead>
<tr>
<th>Original image</th>
<th>Ground truth</th>
<th>σ-Lee filter</th>
<th>Superpixel</th>
<th>Mincut pruning</th>
</tr>
</thead>
</table>

Segmentation / Speckle filtering
Result on real images

Original image

Mincut pruning $\lambda = 10$

858 regions

1372 regions

Mincut pruning $\lambda = 30$

325 regions

579 regions
Outline

- SAR and Polarimetric SAR images
- Tree representation of images
  - Maxtree/mintree
  - Binary Partition Tree
- Tree pruning strategies
  - Pruning techniques for PolSAR images
  - Application to segmentation and speckle filtering
- Tree representation and graph signal processing
  - Filtering strategies
  - Application to ship detection on SAR images.
- Summary and Conclusions
In the case of Polarimetric image segmentation with BPT, the tree node were populated with some homogeneity attributes and the attributes values were used to define the optimal pruning.

Can attribute values be considered as a signal whose support is the tree itself?

Lead to **Graph Signal Processing**.

Illustration: Ship detection in SAR images.

Intuition: Ships have...
- a high gray level value (radiometric attribute)
- an elliptical shape (geometrical attribute)
- Let us capture these features in the maxtree representation.
Processing Strategy

Tree construction → Attribute signal: extraction and processing → Aggregation → Likelihood signal processing → Decision
Graph signal processing

Classical tree representations are inappropriate to observe graph signals.

Even sophisticated graph drawing have limited success (graphviz scalable force directed placement (sfdp)).
Graph signal processing: Branch representation

Original image

Maxtree

Branch representation (2D)

Branches

Gray Level

Distance from root

Branches

Branch representation (3D)
Graph signal processing: Attribute

Ships have a high gray level value:
Gray level graph signal
Graph signal processing: Attribute

Ships have an elliptical shape: **Eccentricity graph signal**

**Attribute**: the eccentricity of the ellipse that has the same second-moments as the connected component represented by the maxtree node.

Random fluctuations of the attribute values along the tree branches -> **need of filtering**
Graph signal processing: Filters

**Graph filter:**
- For each node, define the set of K-hop neighboring nodes and
- Apply a classical filter on the set of nodes

Example of graph attribute (represented by the node gray level values)

Tree are particular graphs but the connectivity towards ancestors may be different from the one towards descendant.
Graph signal processing: Filters

Tree filter:
• The neighborhood of a node is exclusively composed of all its descendants and all its ancestors that are at distance lower or equal to a given value (not the descendants of the ancestors).

Tree median filter of size 2

Graph median filter of size 2
Graph signal processing: Filters

**Tree filter:**
- The neighborhood of a node is exclusively composed of **all its descendants** and **all its ancestors** that are at distance lower or equal to a given value (not the descendants of the ancestors).

The disparity of nodes may lead to unintuitive filtering results (ex: the value node 1 is largely dominated by node 2 and its numerous descendants).
**Branch filter:**

- Two step approach: Estimation and aggregation.
- Estimation: Collect the attribute values on each branch and apply a filter.
- Aggregation: Filter the values obtained for each branch and define the final value.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Estimation</th>
<th>Aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch Mean</td>
<td>1D Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>Branch Median</td>
<td>1D Median</td>
<td>Median</td>
</tr>
<tr>
<td>Branch Erosion</td>
<td>1D Erosion</td>
<td>Min</td>
</tr>
<tr>
<td>Branch Dilation</td>
<td>1D Dilation</td>
<td>Max</td>
</tr>
</tbody>
</table>

**TABLE I**

*Definition of the estimation and the aggregation steps for elementary branch filters*
Graph signal processing: Filters

In some cases the distinction disappears:

- Graph and Tree filters of size one are equivalent.
- Flat Branch and Tree erosion and dilation (opening and closing) are equivalent.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Estimation</th>
<th>Aggregation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch Mean</td>
<td>1D Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>Branch Median</td>
<td>1D Median</td>
<td>Median</td>
</tr>
<tr>
<td>Branch Erosion</td>
<td>1D Erosion</td>
<td>Min</td>
</tr>
<tr>
<td>Branch Dilation</td>
<td>1D Dilation</td>
<td>Max</td>
</tr>
</tbody>
</table>
Graph signal processing: Filters
Ships have an elliptical shape: **Eccentricity graph signal**

**Attribute**: Area ratio

Relation between the area of the connected component and that of the best fitting ellipse

High values on nodes corresponding to ships but also for the root of the tree (image support can be rather well approximated by an ellipse) -> **need of a cleaning process similar to removing maxima that intercept the image border.**
Graph signal processing: Filters

Original: X

Marker: Y

Reconstruction: $\gamma^{rec}(X; Y) = \lim_{k \to \infty} Y_k$

Top-Hat

$X - \gamma^{rec}(X; Y)$

$Y_k = \delta_S(Y_k) \land X$

Area Ratio

Area Ratio (Reconstruction)

Area Ratio (Processed: Top Hat) + Tree Median
Processing Strategy

Aggregation performed by a SVM (estimates the likelihood of a node to represent a ship)
Likelihood estimation with SVM

Presence of three ships

Spurious estimation

Aggregation performed by a SVM (estimates the likelihood of a node to represent a ship)
Representing and processing the attribute maxima with a maxtree.

Likelihood signal processing

Tree1: Original likelihood Maxtree (sfdp representation)

Tree2: Maxtree of Tree1

Pruned Tree2 (Area extinction filter)

Restitution of Tree1 with pruned Tree2
Results

Tree construction

Mean gray level: extraction and processing

Eccentricity: extraction and processing

Aggregation

Likelihood signal processing

Decision

Area ratio: extraction and processing

Node classification (SVM)

Ship classification (Thresholding)

Results Image and Video Processing 58
## Results: node classification

### Training

<table>
<thead>
<tr>
<th>Set</th>
<th>Precision</th>
<th>Recall</th>
<th>F-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training</td>
<td>0.853</td>
<td>0.875</td>
<td>0.864</td>
</tr>
<tr>
<td>Validation</td>
<td>0.840</td>
<td>0.868</td>
<td>0.853</td>
</tr>
<tr>
<td>Test</td>
<td>0.828</td>
<td>0.892</td>
<td>0.858</td>
</tr>
</tbody>
</table>

No overtraining

### Node classification results without attribute filtering

<table>
<thead>
<tr>
<th></th>
<th>Graph filters</th>
<th>Tree filters</th>
<th>Branch filters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Opening</td>
</tr>
<tr>
<td></td>
<td>Precision</td>
<td>Recall</td>
<td>F-Score</td>
</tr>
<tr>
<td>Training</td>
<td>0.918</td>
<td>0.861</td>
<td>0.889</td>
</tr>
<tr>
<td>Validation</td>
<td>0.915</td>
<td>0.866</td>
<td>0.890</td>
</tr>
<tr>
<td>Test</td>
<td>0.913</td>
<td>0.874</td>
<td>0.893</td>
</tr>
<tr>
<td></td>
<td>Precision</td>
<td>Recall</td>
<td>F-Score</td>
</tr>
<tr>
<td>Training</td>
<td>0.884</td>
<td>0.872</td>
<td>0.878</td>
</tr>
<tr>
<td>Validation</td>
<td>0.863</td>
<td>0.869</td>
<td>0.866</td>
</tr>
<tr>
<td>Test</td>
<td>0.887</td>
<td>0.890</td>
<td>0.888</td>
</tr>
<tr>
<td></td>
<td>Precision</td>
<td>Recall</td>
<td>F-Score</td>
</tr>
<tr>
<td>Training</td>
<td>0.905</td>
<td>0.881</td>
<td>0.893</td>
</tr>
<tr>
<td>Validation</td>
<td>0.906</td>
<td>0.874</td>
<td>0.890</td>
</tr>
<tr>
<td>Test</td>
<td>0.886</td>
<td>0.871</td>
<td>0.878</td>
</tr>
<tr>
<td></td>
<td>Precision</td>
<td>Recall</td>
<td>F-Score</td>
</tr>
<tr>
<td>Training</td>
<td>0.844</td>
<td>0.861</td>
<td>0.872</td>
</tr>
<tr>
<td>Validation</td>
<td>0.887</td>
<td>0.871</td>
<td>0.879</td>
</tr>
<tr>
<td>Test</td>
<td>0.867</td>
<td>0.866</td>
<td>0.866</td>
</tr>
</tbody>
</table>

Node classification results with attribute filtering

Shows the interest of attribute graph signal processing and of going beyond classical graph filters.
Results: Ship classification

<table>
<thead>
<tr>
<th>Ship detection approach</th>
<th>Precision</th>
<th>Recall</th>
<th>F-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed approach without area extinction filter</td>
<td>0.878</td>
<td>1.000</td>
<td>0.935</td>
</tr>
<tr>
<td>Proposed approach with area extinction filter</td>
<td>0.947</td>
<td>1.000</td>
<td><strong>0.973</strong></td>
</tr>
<tr>
<td>CFAR approach [1]</td>
<td>0.865</td>
<td>0.889</td>
<td>0.877</td>
</tr>
<tr>
<td>Wavelet-based approach [2]</td>
<td>0.857</td>
<td>0.923</td>
<td>0.889</td>
</tr>
<tr>
<td>GLRT approach [3]</td>
<td>0.872</td>
<td>0.944</td>
<td>0.907</td>
</tr>
<tr>
<td>Entropy-based dissimilarity [4]</td>
<td>0.800</td>
<td>1.000</td>
<td>0.889</td>
</tr>
</tbody>
</table>

Shows the interest of the area extinction filter processing the likelihood information.
## Results: Ship classification

<table>
<thead>
<tr>
<th>Ship detection approach</th>
<th>Precision</th>
<th>Recall</th>
<th>F-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed approach without area extinction filter</td>
<td>0.878</td>
<td>1.000</td>
<td>0.935</td>
</tr>
<tr>
<td>Proposed approach with area extinction filter</td>
<td>0.947</td>
<td>1.000</td>
<td><strong>0.973</strong></td>
</tr>
<tr>
<td>CFAR approach [1]</td>
<td>0.865</td>
<td>0.889</td>
<td>0.877</td>
</tr>
<tr>
<td>Wavelet-based approach [2]</td>
<td>0.857</td>
<td>0.923</td>
<td>0.889</td>
</tr>
<tr>
<td>GLRT approach [3]</td>
<td>0.872</td>
<td>0.944</td>
<td>0.907</td>
</tr>
<tr>
<td>Entropy-based dissimilarity [4]</td>
<td>0.800</td>
<td>1.000</td>
<td>0.889</td>
</tr>
</tbody>
</table>


Conclusions

• **Show the interest of processing images from a structured representation of its pixels:**
  • Hierarchical representation leading to trees: Max/Mintree and BPT as examples

• **Processing strategies:**
  • Act on the support of the tree: Pruning
    • Useful for segmentation and for filtering (connected operators)
  • Populate the tree with attributes values and treat these attribute values as graph signals
    • Filtering tools: Graph, Tree and Branch versions
    • Morphological reconstruction
    • Connected filters on the graph signal (either by reconstruction or by tree representation).
  • Illustrate the interest for object detection
Thank you for your attention

More information:


- Matlab toolbox: https://github.com/imatge-upc/Maxtree-Processing-Toolbox