
Processing Radar Images with Hierarchical Region-Based Representations and Graph Signal Processing Tools

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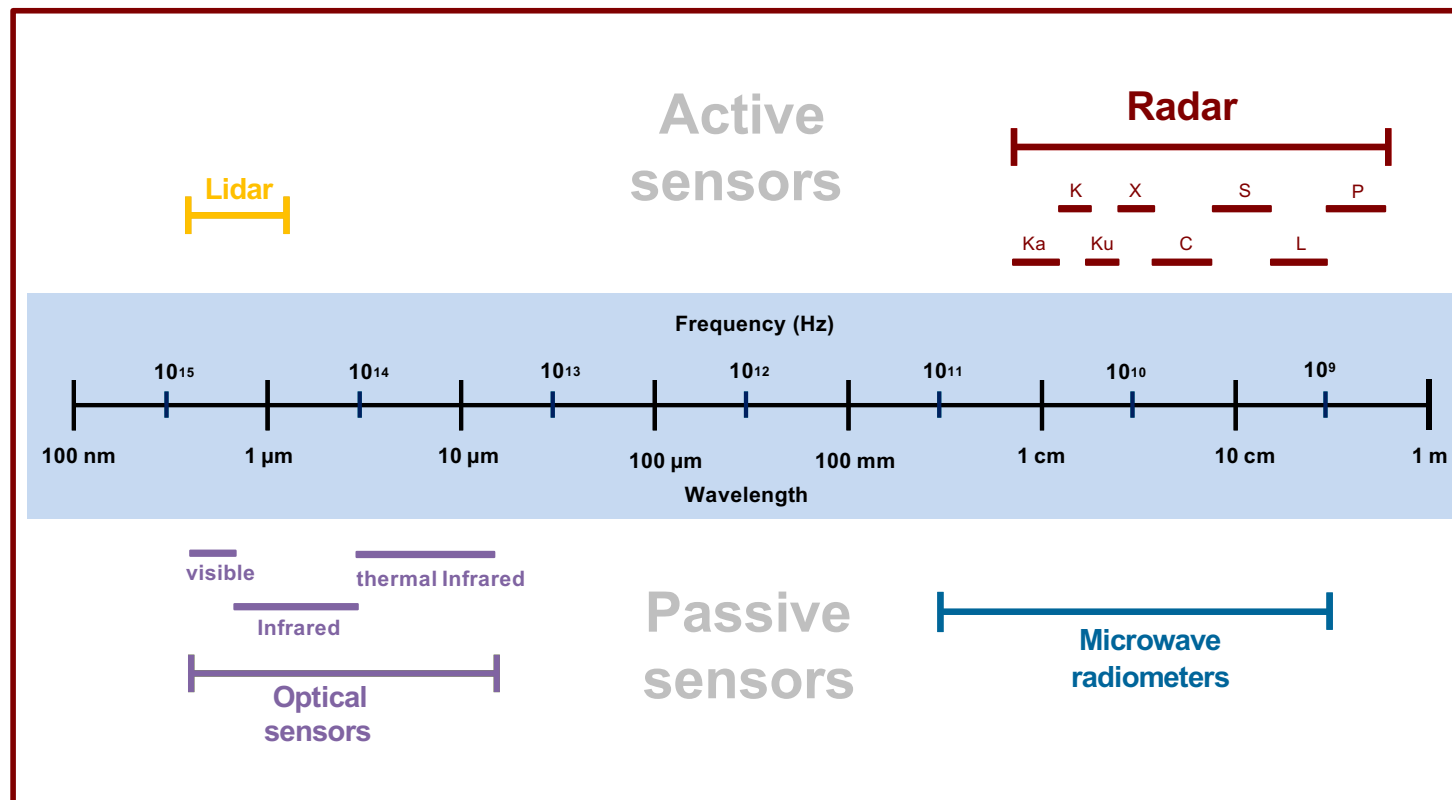
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Outline

- **SAR and Polarimetric SAR images**
- **Tree representation of images**
 - Maxtree/mintree
 - Binary Partition Tree
- **Tree pruning strategies**
 - Pruning techniques for PolSAR images
 - Application to segmentation and speckle filtering
- **Tree representation and graph signal processing**
 - Filtering strategies
 - Application to ship detection on SAR images.
- **Summary and Conclusions**

SAR and Polarimetric SAR Images



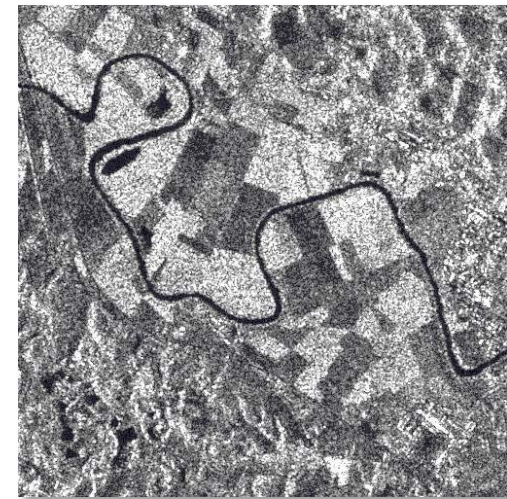
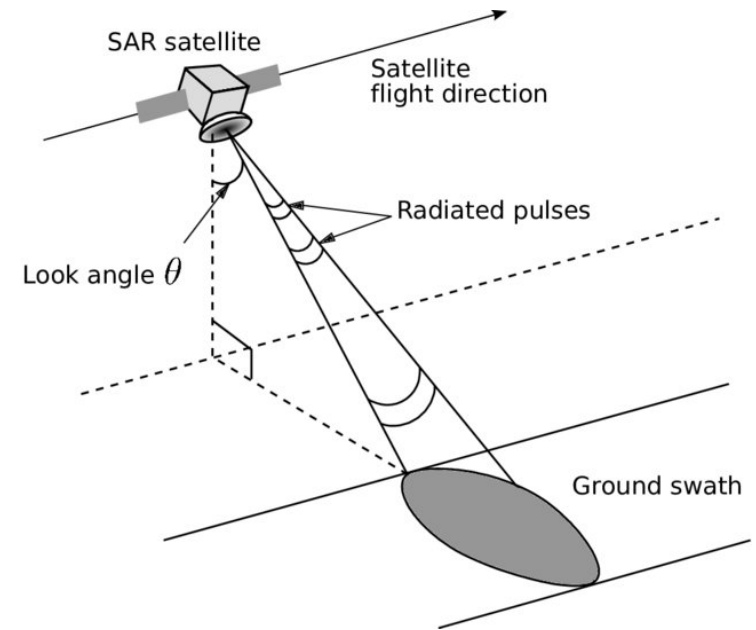
Radar remote sensing:

- Complementary information to optical systems
- Penetration of radar waves (high penetration for long wavelengths, allows volume modeling)
- Weather independent & day-and-night imaging capability

SAR and Polarimetric SAR Images

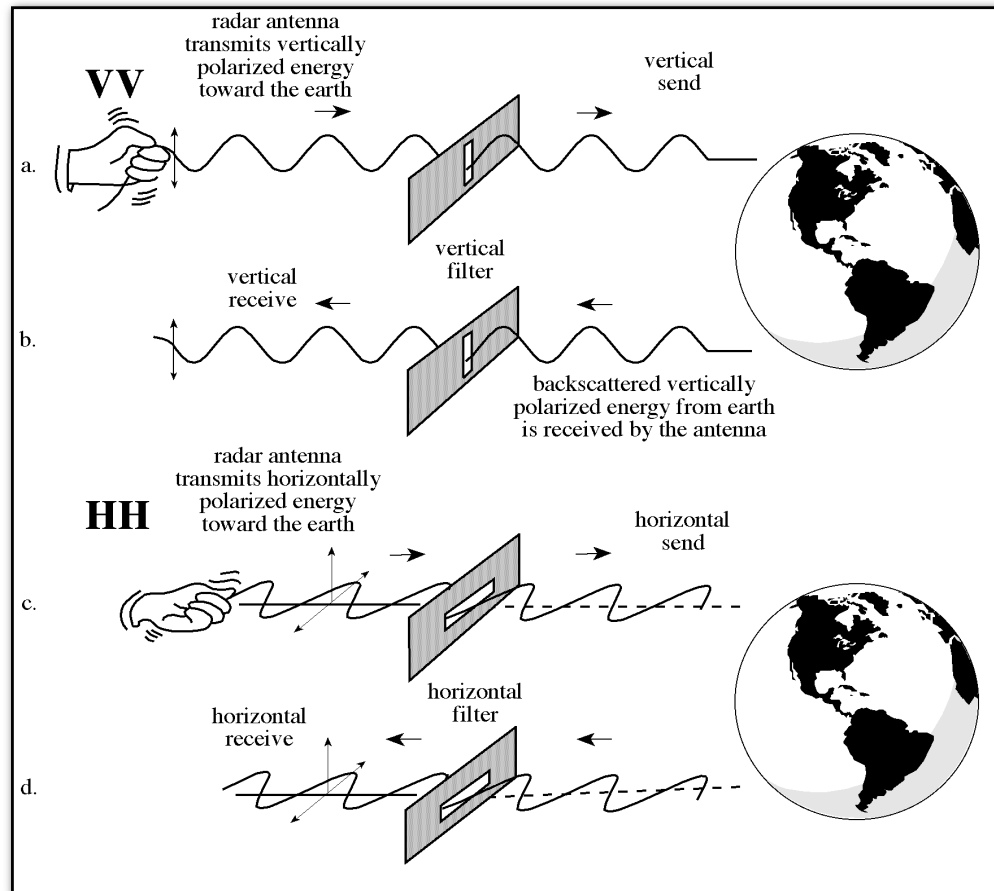
SAR (Synthetic aperture radar) images:

- Electromagnetic waves are sequentially transmitted and **backscattered echoes** are recorded while the **platform is moving**.
- The coherent signal combination allows the construction of a **virtual aperture** much **longer than the physical antenna** length.
- Measure the **scene reflectivity** and SAR images are commonly displayed in terms of intensity values.
- Images are corrupted by **speckle noise** (presence of many elemental scatterers with a random distribution within a resolution cell. The coherent sum results in strong fluctuations of the backscattering)



Polarimetric SAR images

- Add diversity
- Acquisition of polarization states of an electromagnetic field (sensitive to the target geometry and the dielectric properties of targets)

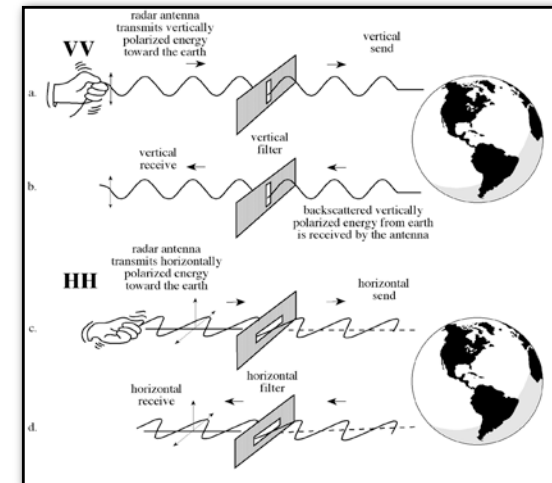


Polarimetric SAR images

- PolSAR images:**

- The measured information is the scattering matrix:

$$S_c = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \xrightarrow{\text{Vectoriz.}} \begin{bmatrix} S_{hh} & \sqrt{2}S_{hv} & S_{vv} \end{bmatrix}$$



- Each PolSAR pixel is represented by a covariance matrix (under the Gaussian scattering assumption):

$$C = \begin{bmatrix} E\{|S_{hh}|^2\} & \sqrt{2}E\{S_{hh}S_{hv}^*\} & E\{S_{hh}S_{vv}^*\} \\ \sqrt{2}E\{S_{hv}S_{hh}^*\} & 2E\{|S_{hv}|^2\} & \sqrt{2}E\{S_{hv}S_{vv}^*\} \\ E\{S_{vv}S_{hh}^*\} & \sqrt{2}E\{S_{vv}S_{hv}^*\} & E\{|S_{vv}|^2\} \end{bmatrix}$$

Image representation

- **Images** are classically seen as a **set of pixels**, but pixels....
 - are **very numerous**,
 - carry a **small amount of information**,
 - are **unstructured**,
 - are heavily **corrupted by speckle noise** in the (Pol)SAR case.

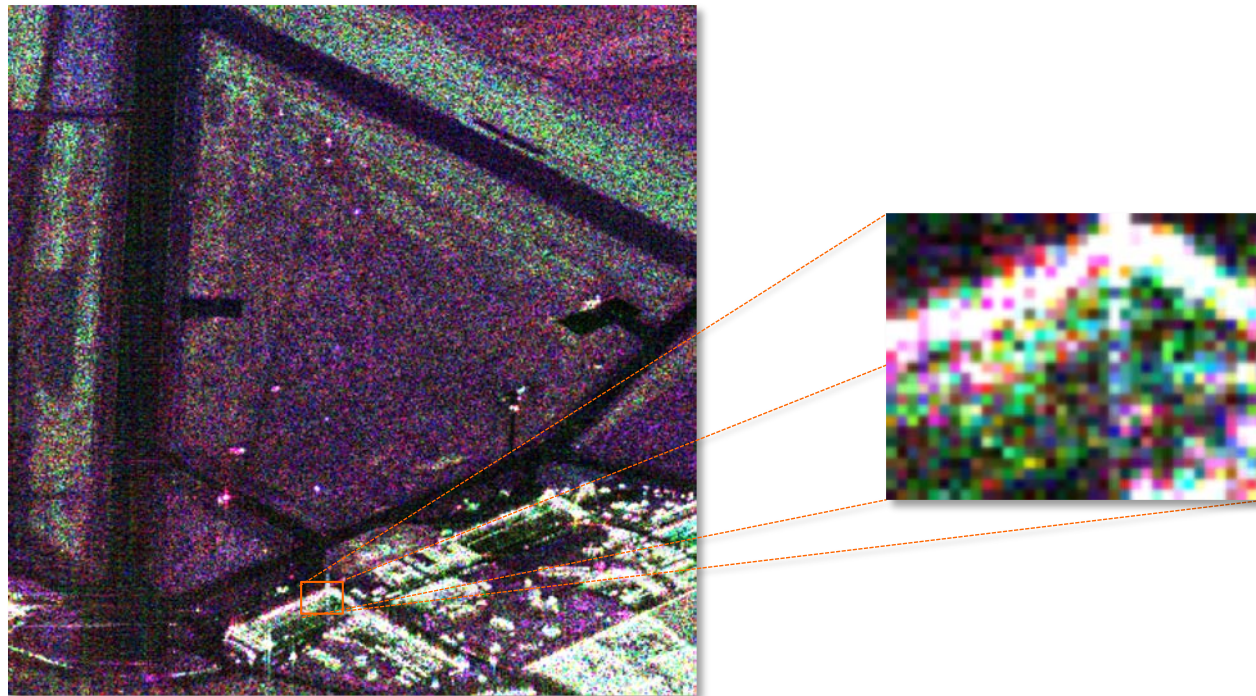


Image representation

We would like an image representation....

- Made of more meaningful primitives,
- Involving a reduced number of primitives.

=> **Need of pixel aggregation**

The representation should support many different applications....
But, the relevant scale for the application is unknown.

=> **Need of multiscale representation**

Easy access to the information

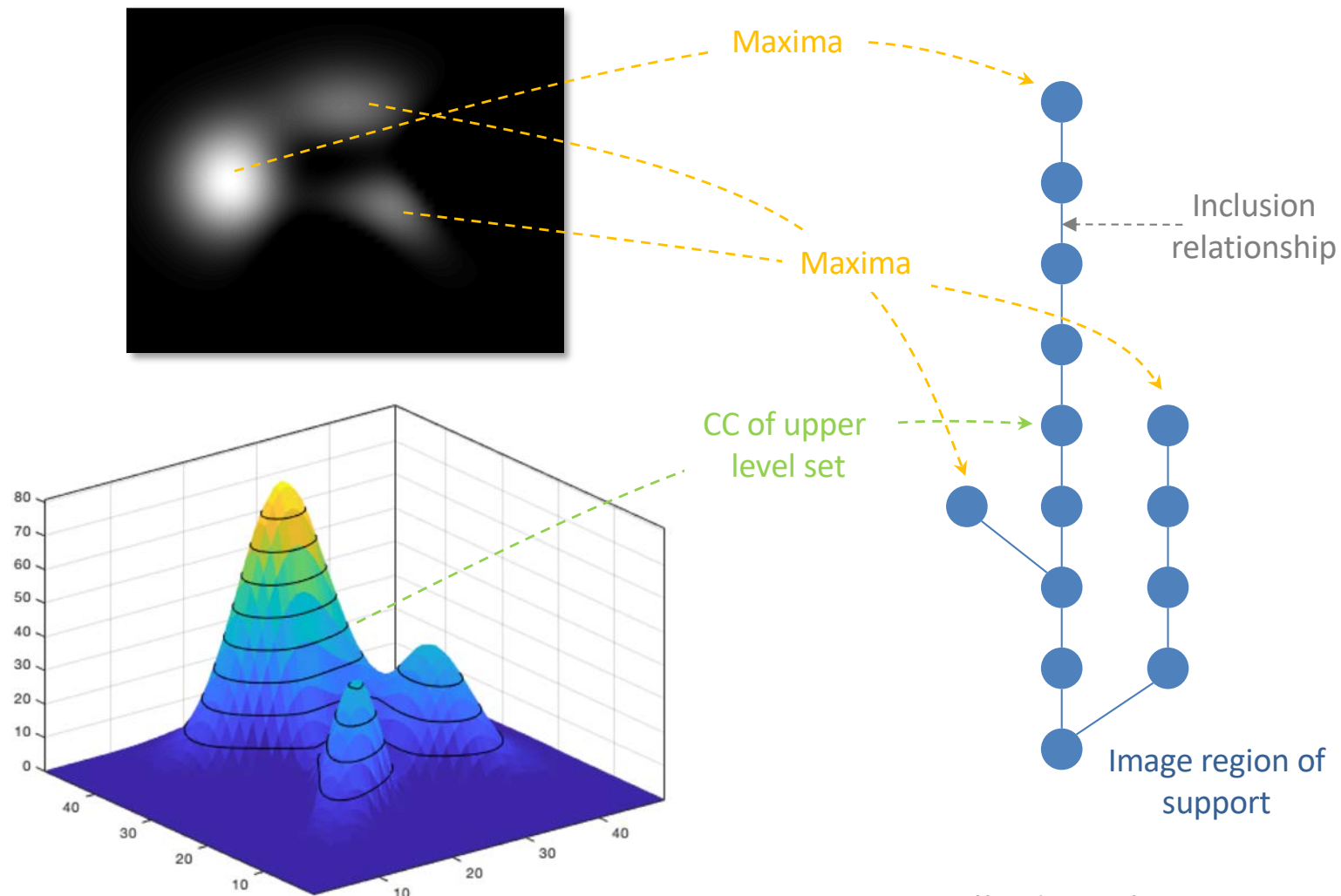
=> **Need of structured representation**

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Maxtree/Mintree

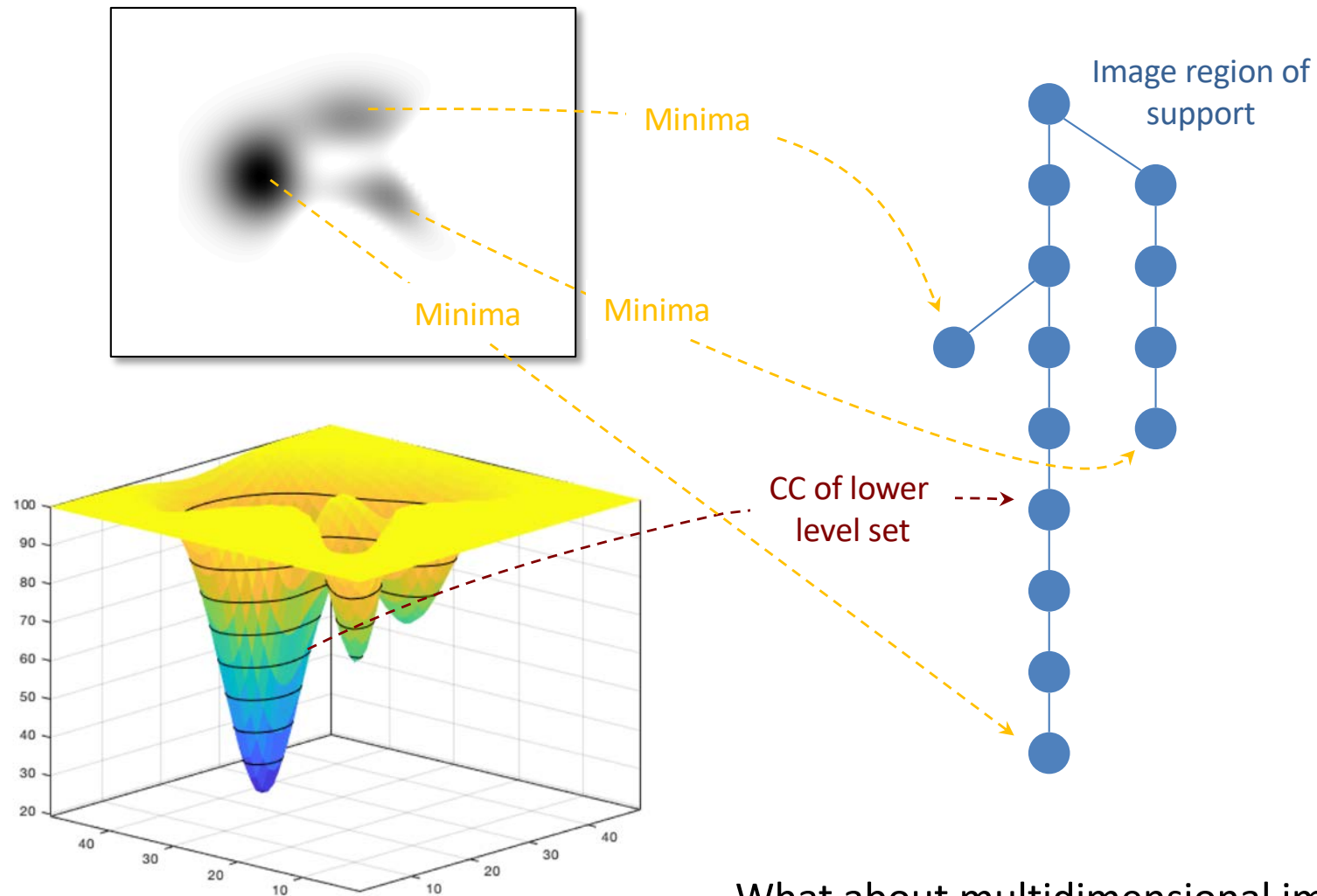
Maxtree: Structuring of the binary connected components of upper level sets



Essentially describes maxima

Maxtree/Mintree

Mintree: Dual representation (lower level sets), describe minima

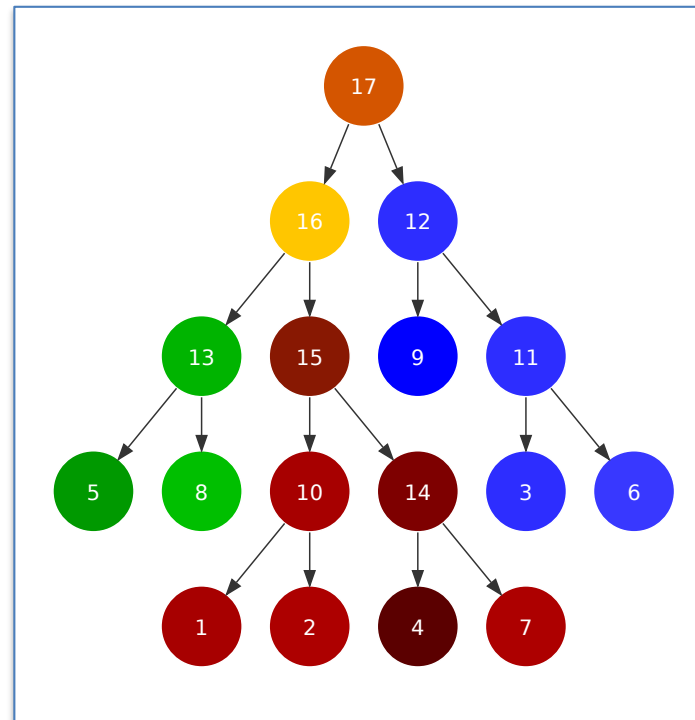
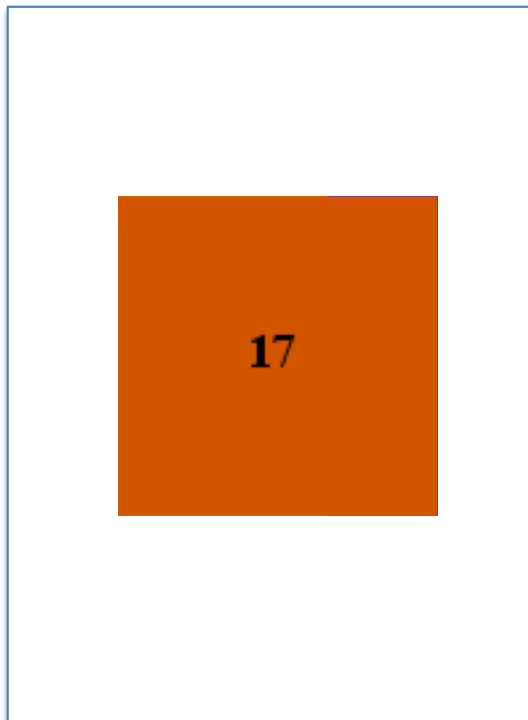


What about multidimensional images?

Binary Partition Tree

Iterative region merging algorithm

- Start with an initial partition (pixels or superpixel partition)
- Iteratively merge the most similar pair of neighboring regions

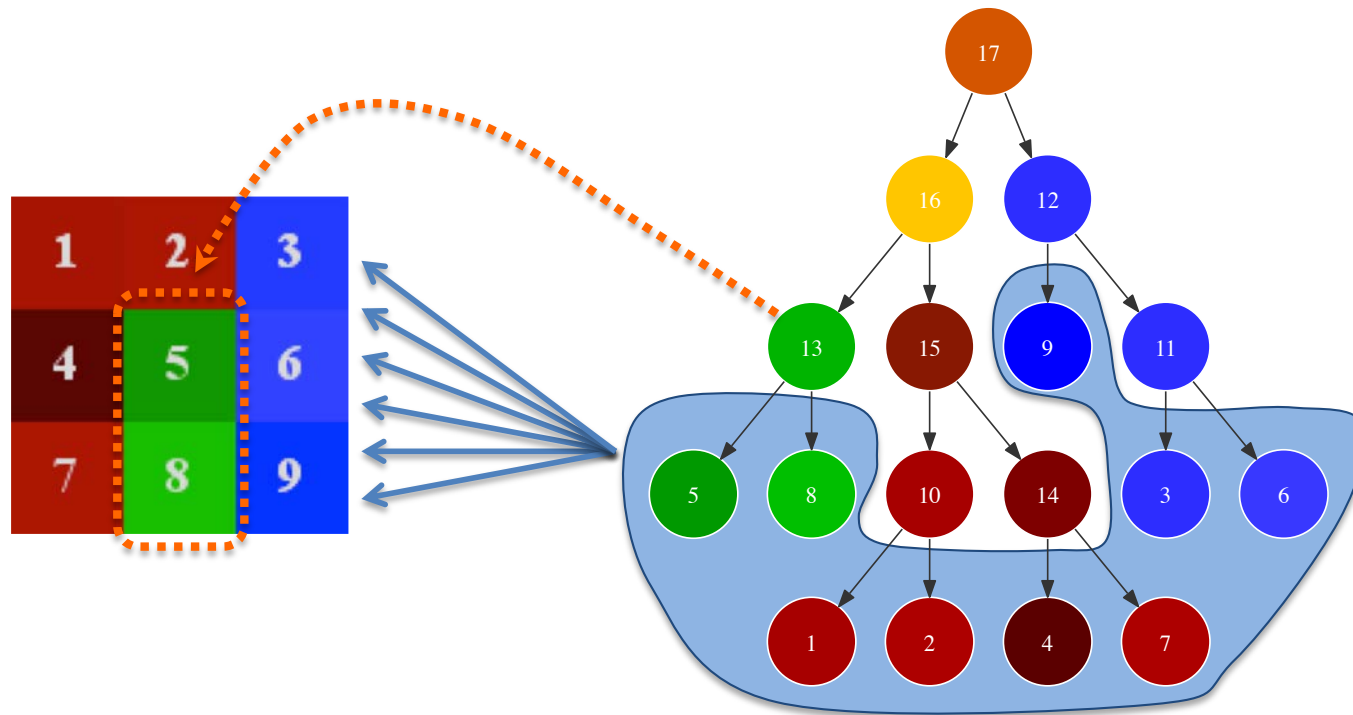


Essentially as a hierarchical segmentation but **keep track of the merging sequence in a tree structure.**

Binary Partition Tree

BPT: Hierarchical Region-based representation where

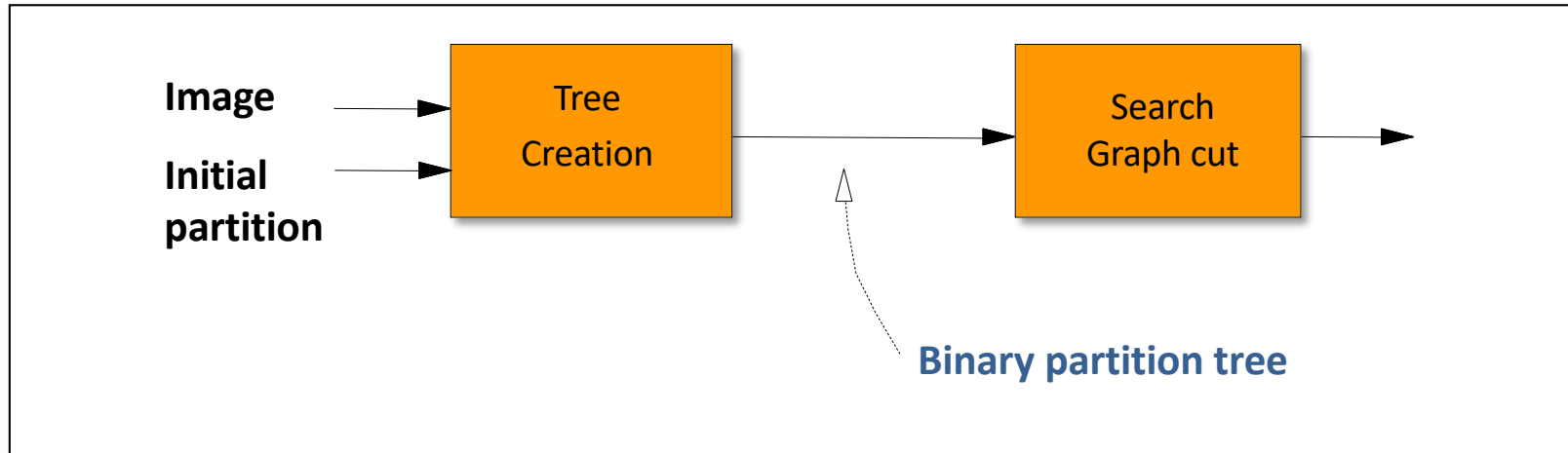
- **Leaves** represent the image pixels or regions of an initial (oversegmented) partition
- The **remaining nodes** represent the union of child nodes
- The **links** represent the inclusion relationship



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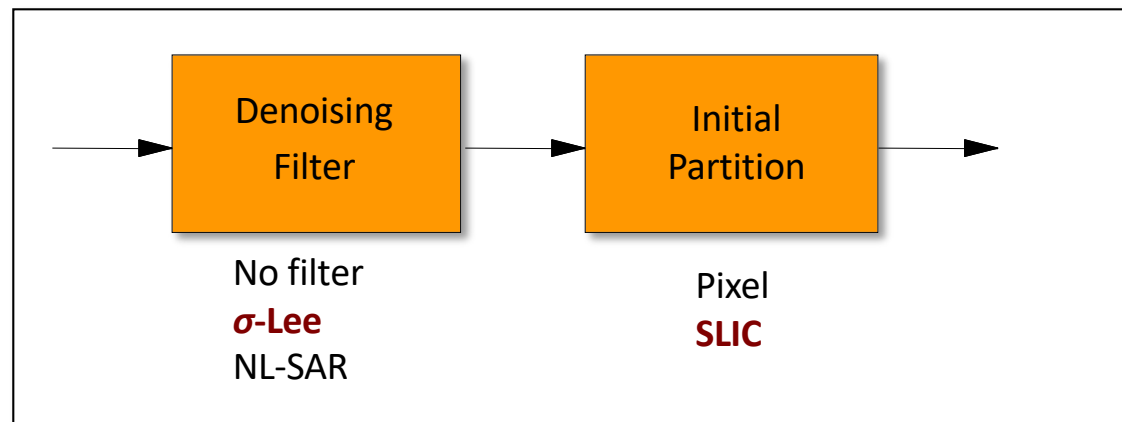
Image Processing with BPT



BPT construction for PolSAR images

Initial partition for PolSAR images

- It is necessary to pre-filter the data to deal with the speckle noise
- The use of super-pixel partitions provides a drastic reduction of the complexity without significant loss in quality



Iterative region merging algorithm

- Start with the super-pixel partition
- Iteratively merge the most similar pair of neighboring regions

P. Salembier, S. Foucher, and C. Lopez-Martinez. Low-level processing of PolSAR images with binary partition trees. In IEEE Int. Geoscience and Remote Sensing Symposium, IGARSS 2014, Quebec, Canada, July 2014.

BPT construction for PolSAR images

- **Definition of the merging algorithm:**

- How to model regions:

$$M(R) \quad M(R_1 \cup R_2)$$

- What is the similarity between two neighboring regions:

$$S(R_1, R_2)$$

BPT construction for PolSAR images

- **Definition of the merging algorithm:**

- How to model regions: $M(R) \quad M(R_1 \cup R_2)$

- What is the similarity between two neighboring regions:

$$S(R_1, R_2)$$

- **Region Model:**

- In homogeneous areas, spatial averaging is the MLE under the Gaussian assumption. The region model is therefore defined as the region mean:

$$M(R) = Z_R = \begin{bmatrix} \overline{\{|S_{hh}|^2\}} & \sqrt{2}\overline{\{S_{hh}S_{hv}^*\}} & \overline{\{S_{hh}S_{vv}^*\}} \\ \sqrt{2}\overline{\{S_{hv}S_{hh}^*\}} & 2\overline{\{|S_{hv}|^2\}} & \sqrt{2}\overline{\{S_{hv}S_{vv}^*\}} \\ \overline{\{S_{vv}S_{hh}^*\}} & \sqrt{2}\overline{\{S_{vv}S_{hv}^*\}} & \overline{\{|S_{vv}|^2\}} \end{bmatrix}$$

BPT construction for PolSAR images

- **Region Similarity:**

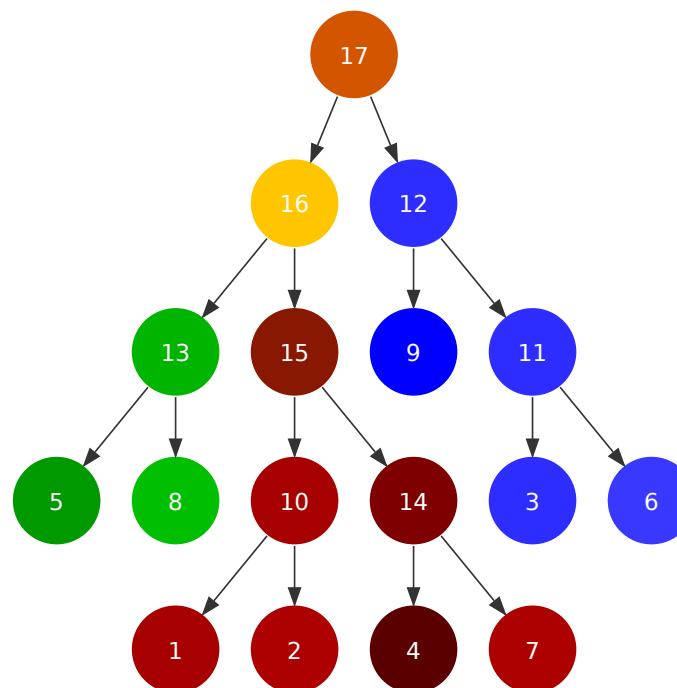
- If pixels were carrying color information in the *Lab* space, we could use the Euclidean distance as similarity, but....
- Similarity between covariance matrices: **Geodesic distance** in the manifold of hermitian positive definite matrices:

$$S(R_1, R_2) = \underbrace{\left\| \log(Z_{R_1}^{-1/2} Z_{R_2} Z_{R_1}^{-1/2}) \right\|}_{\text{Geodesic distance}} \cdot \underbrace{\ln(2n_1 n_2 / (n_1 + n_2))}_{\text{Encourage the merging of small regions}}$$

- [3] A. Alonso-González, S. Valero, J. Chanussot, C. López-Martínez and P. Salembier, “Processing Multidimensional SAR and Hyperspectral Images With Binary Partition Tree”, *Proceedings of the IEEE*, vol. 101, no. 3, pp. 723 – 747, 2013.

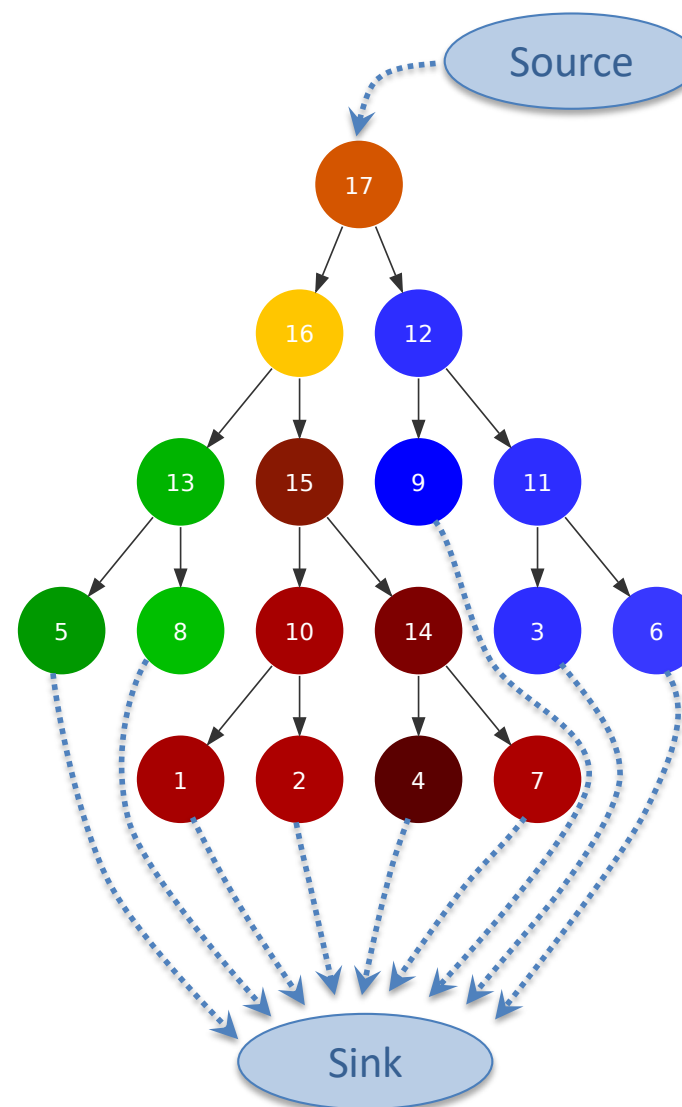
BPT Processing

- **Extracting a partition from a BPT**
 - Simplest approach: Stop the merging at a given point (merging sequence truncation).
 - Partitions can also be extracted by a particular graph cut: **pruning**.



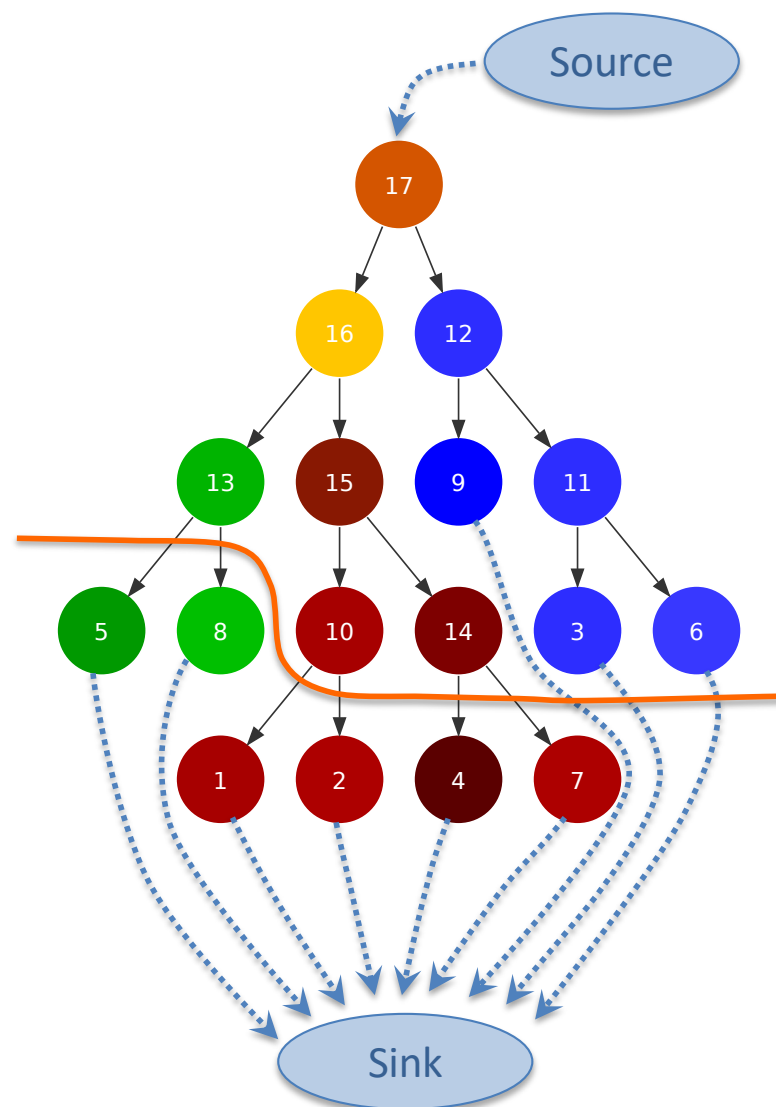
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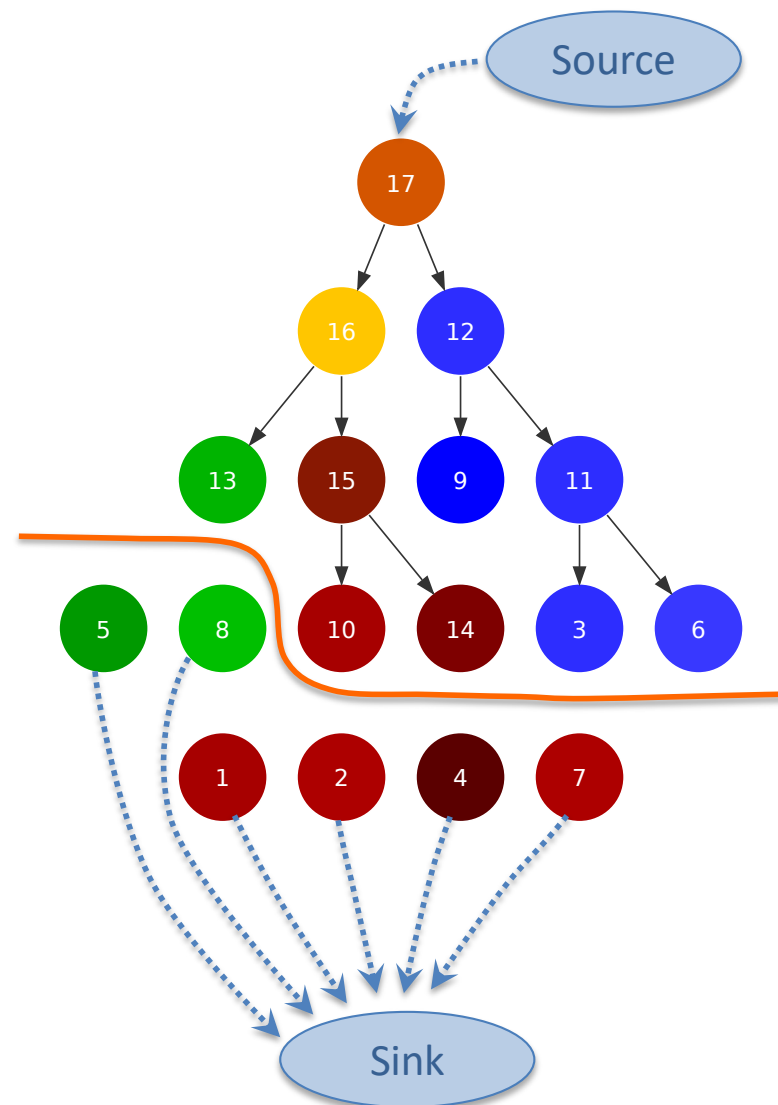
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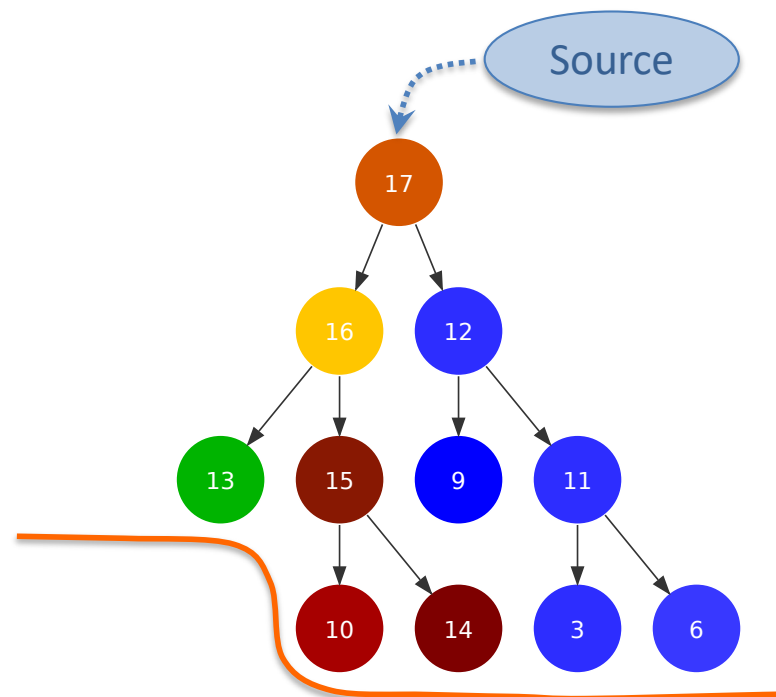
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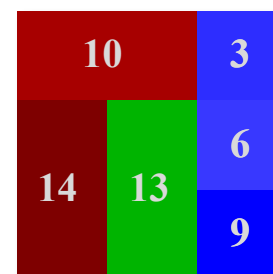
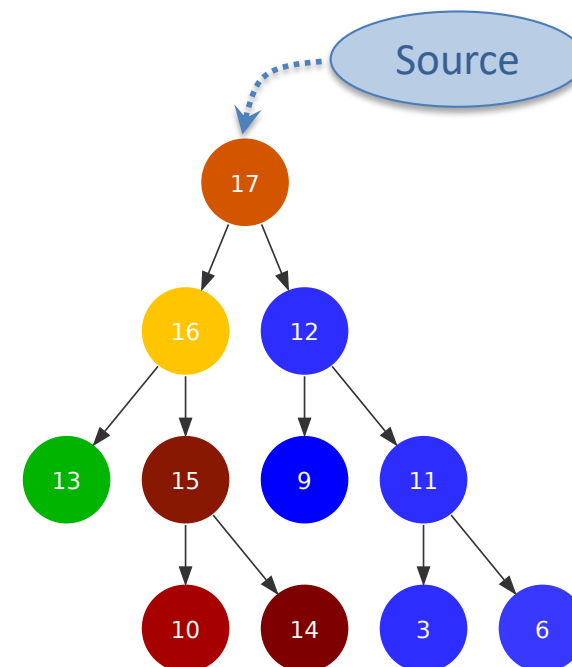
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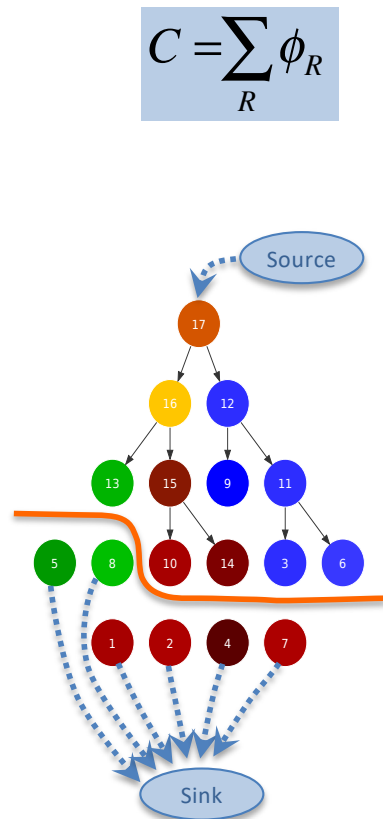
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Not seen during the merging sequence

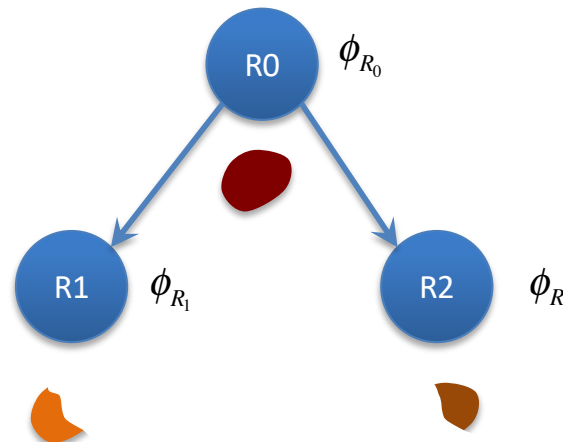
Optimum pruning technique

- The optimization of an h-increasing criterion (in particular additive) on the tree can be efficiently done through dynamic programming.



$$C = \sum_R \phi_R$$

Local decisions are globally optimum



if $\phi_{R_0} < \phi_{R_1} + \phi_{R_2}$
Select R_0
else
Select R_1 & R_2

J. Serra. Hierarchy and optima. In Debled-Renneson et al., editor, Discrete Geometry for Computer Imagery, LNCS 6007, Springer, 2011.

1) Squared Error

- Define a homogeneity criterion: $C = \sum_R \phi_R$ with $\phi_R = \sum_{i,j \in R} \|Z_{i,j}^I - Z_R\|$

Pruning criteria

1) Squared Error

- Define a homogeneity criterion: $C = \sum_R \phi_R$ with $\phi_R = \sum_{i,j \in R} \|Z_{i,j}^I - Z_R\|$
- Issue:** The criterion has to be regularized:
$$\phi_R^{SE} = \sum_{i,j \in R} \|Z_{i,j}^I - Z_R\| + \lambda$$

Data fidelity Regularization

Pruning criteria

1) Squared Error

- Define a homogeneity criterion: $C = \sum_R \phi_R$ with $\phi_R = \sum_{i,j \in R} \|Z_{i,j}^I - Z_R\|$

- Issue:** The criterion has to be regularized:

$$\phi_R^{SE} = \sum_{i,j \in R} \|Z_{i,j}^I - Z_R\| + \lambda$$

Data fidelity Regularization

2) SAR Squared Error

- Deal with the speckle noise (multiplicative):

$$\phi_R^{SAR_SE} = \sum_{i,j \in R} \frac{\|Z_{i,j}^I - Z_R\|}{\|Z_R\|} + \lambda$$

Pruning criteria

3) Wishart distance

- Similarity between covariance matrices of pixels and regions:

$$tr((Z_{ij}^I)^{-1} Z_R) + tr(Z_R^{-1} Z_{ij})$$

- Issue:** Matrix inversion for each pixel => use only diagonal elements:

$$\phi_R^{Wishart} = \sum_{i,j \in R} \sqrt{\sum_{k=1,2,3} \frac{Z_{ij}^I(k,k)^2 + Z_R(k,k)^2}{Z_{ij}^I(k,k) Z_R(k,k)}} + \lambda$$

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4) Geodesic distance

- Distance adapted to the cone of hermitian positive definite matrices:

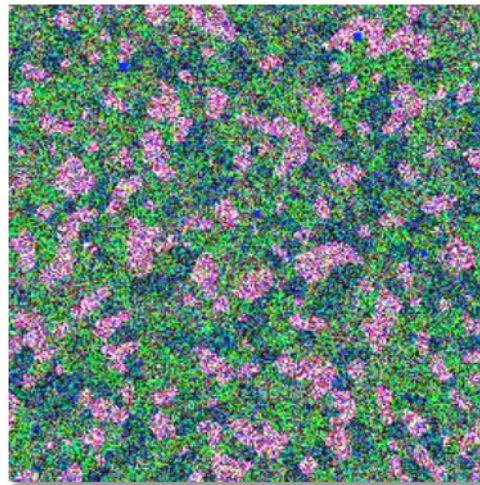
$$\left\| \log(Z_R^{-1/2} Z_{ij}^I Z_R^{-1/2}) \right\|$$

- Complex => use only diagonal elements:

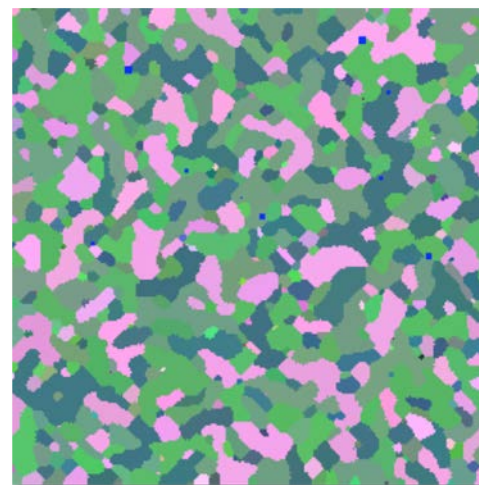
$$\phi_R^{Geodesic} = \sum_{i,j \in R} \sqrt{\sum_{k=1,2,3} \ln^2 \left(\frac{Z_{ij}^I(k,k)}{Z_R(k,k)} \right)} + \lambda$$

BPT pruning evaluation

- Use of a dataset with ground-truth:
 - Simulated PolSAR Images
 - Typical polarimetric responses have been extracted from an AIRSAR image (L-band)
 - Class regions are spatially modeled by a Markov Random Field
 - Single look images have been generated using Cholesky decomposition



Original image



Ground-truth image

RGB-pauli
color coding

S. Foucher and C. Lopez-Martinez. Analysis, evaluation, and comparison of polarimetric SAR speckle filtering techniques. IEEE Trans. Image Processing, 23(4):1751–1764, 2014.

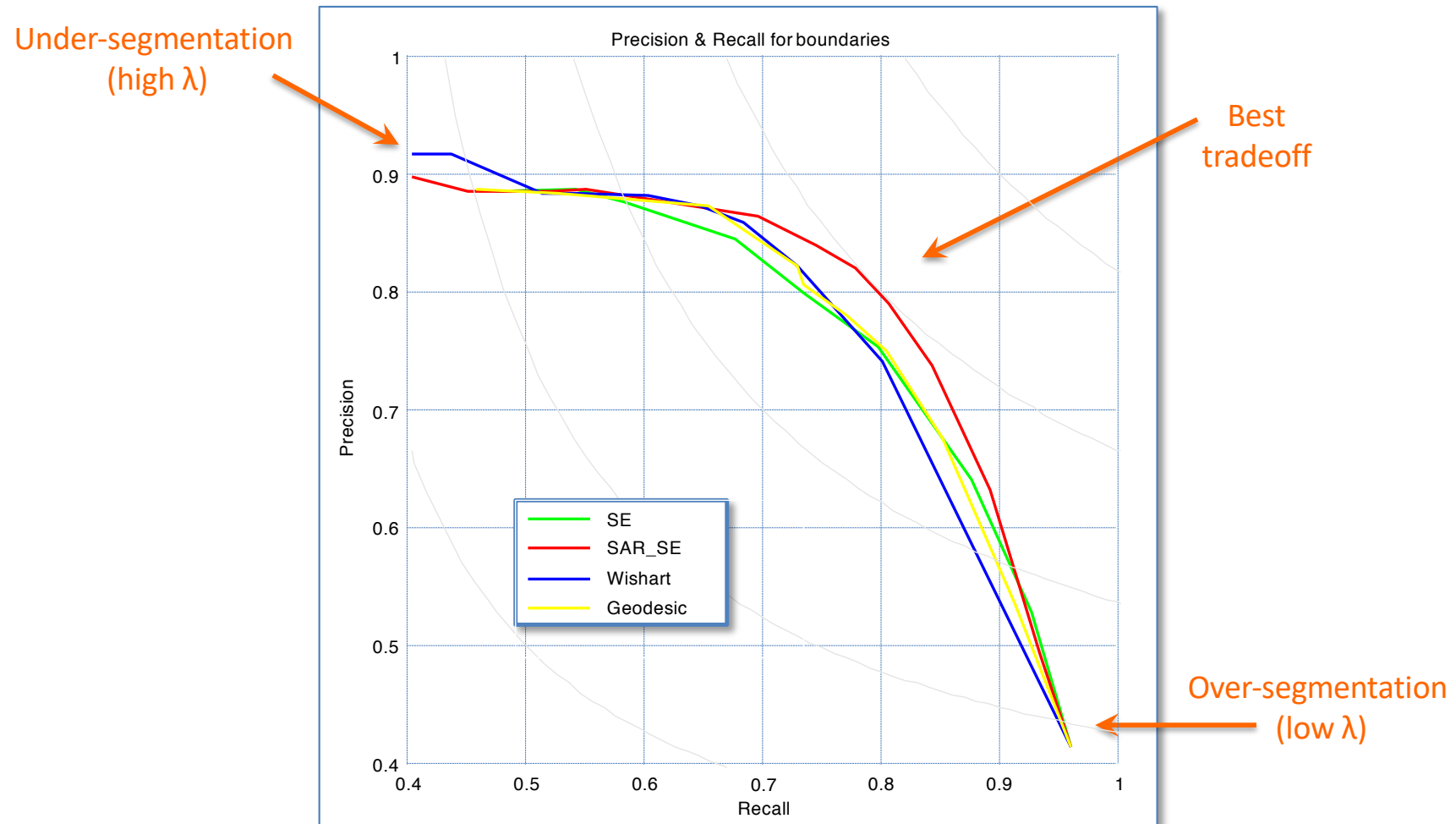
Results

- Assess the quality of the extracted partitions by the **Precision and Recall curves** classically used in computer vision:
 - Classify all pairs of pixels as **boundary** or **non-boundary**
 - Compare the performances of the extracted partition compared to the ground truth

D. Martin, C. Fowlkes, and J. Malik. Learning to detect natural image boundaries using local brightness, color, and texture cues. IEEE Trans. on PAMI, 26(5):530–549, 2004.

- Complete view of the system behavior (from under- to over-segmentation)
- Ideal system should have Precision and Recall close to 1

Results: Precision & Recall curves



Best pruning approach: SAR_SE

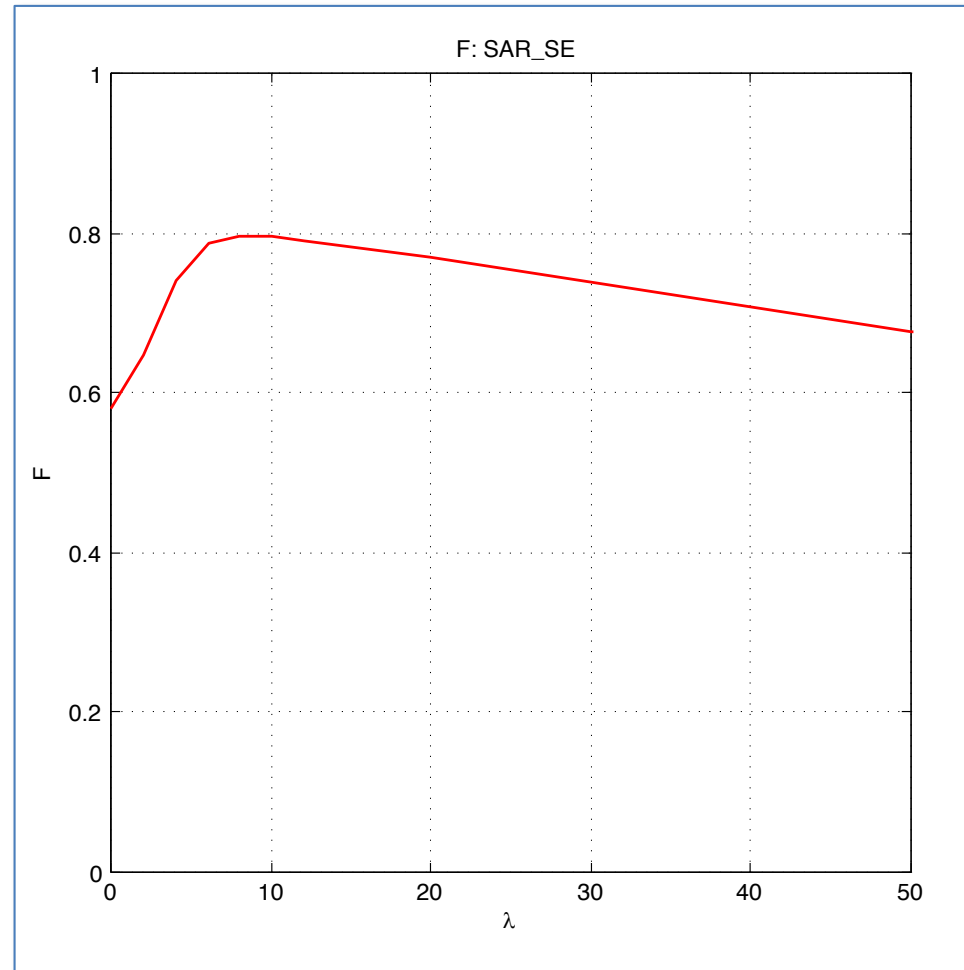
$$\phi_R^{\text{SAR_SE}} = \sum_{i,j \in R} \frac{\|Z_{i,j}^I - Z_R\|}{\|Z_R\|} + \lambda$$

Results: Precision & Recall curves

Sensitivity of the results with respect to the regularization parameter:

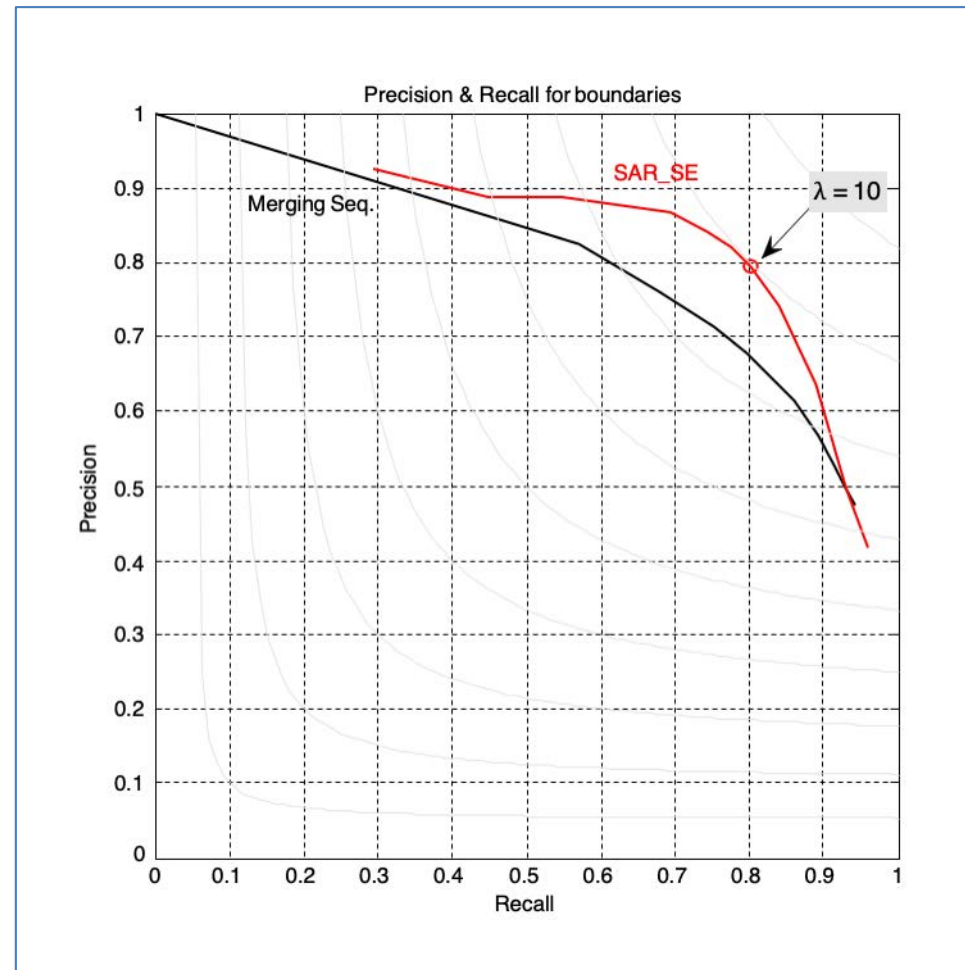
Summary of Precision and Recall with harmonic mean:

$$F = \frac{2PR}{P + R}$$



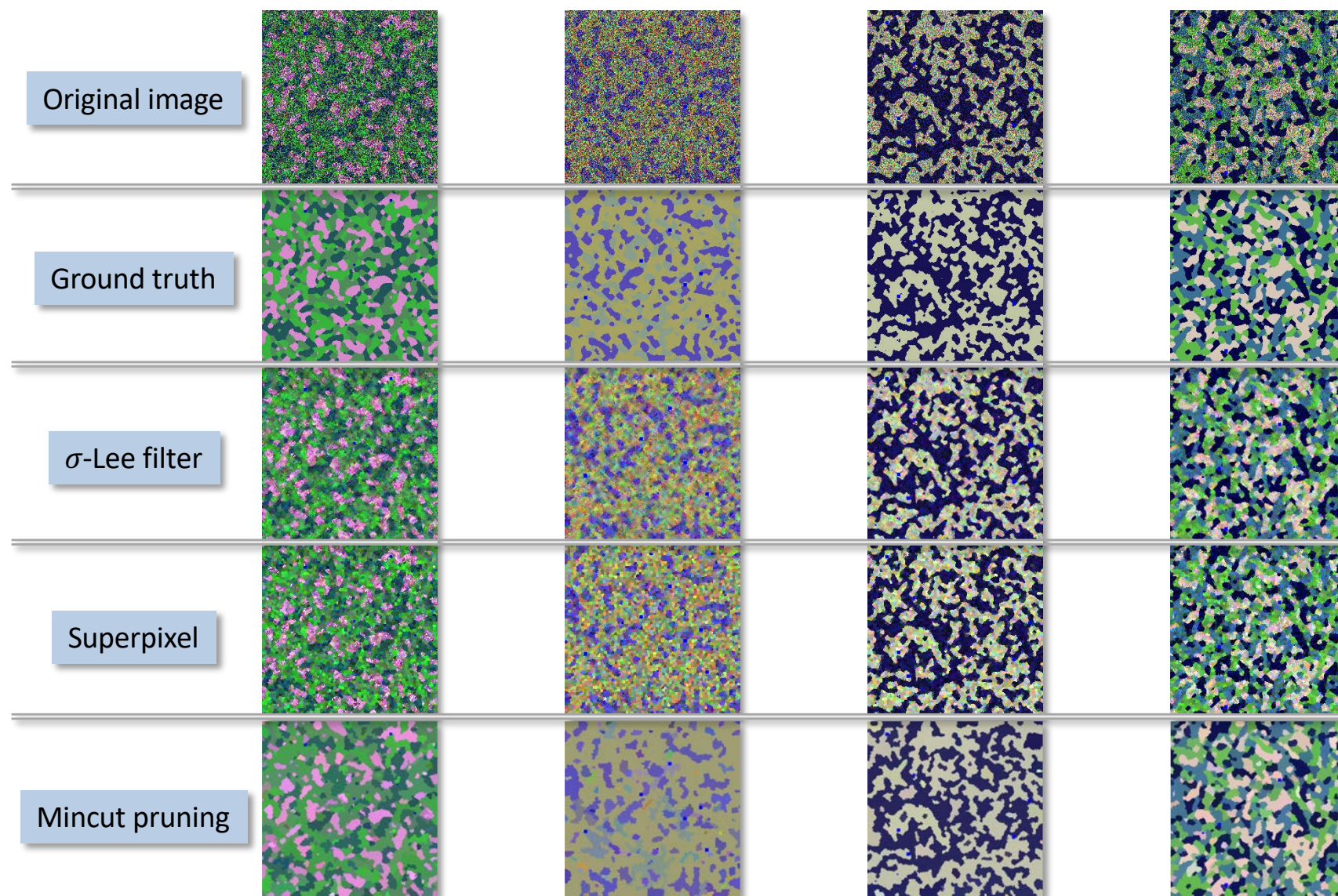
Results: Precision & Recall curves

Merging sequence truncation versus Mincut



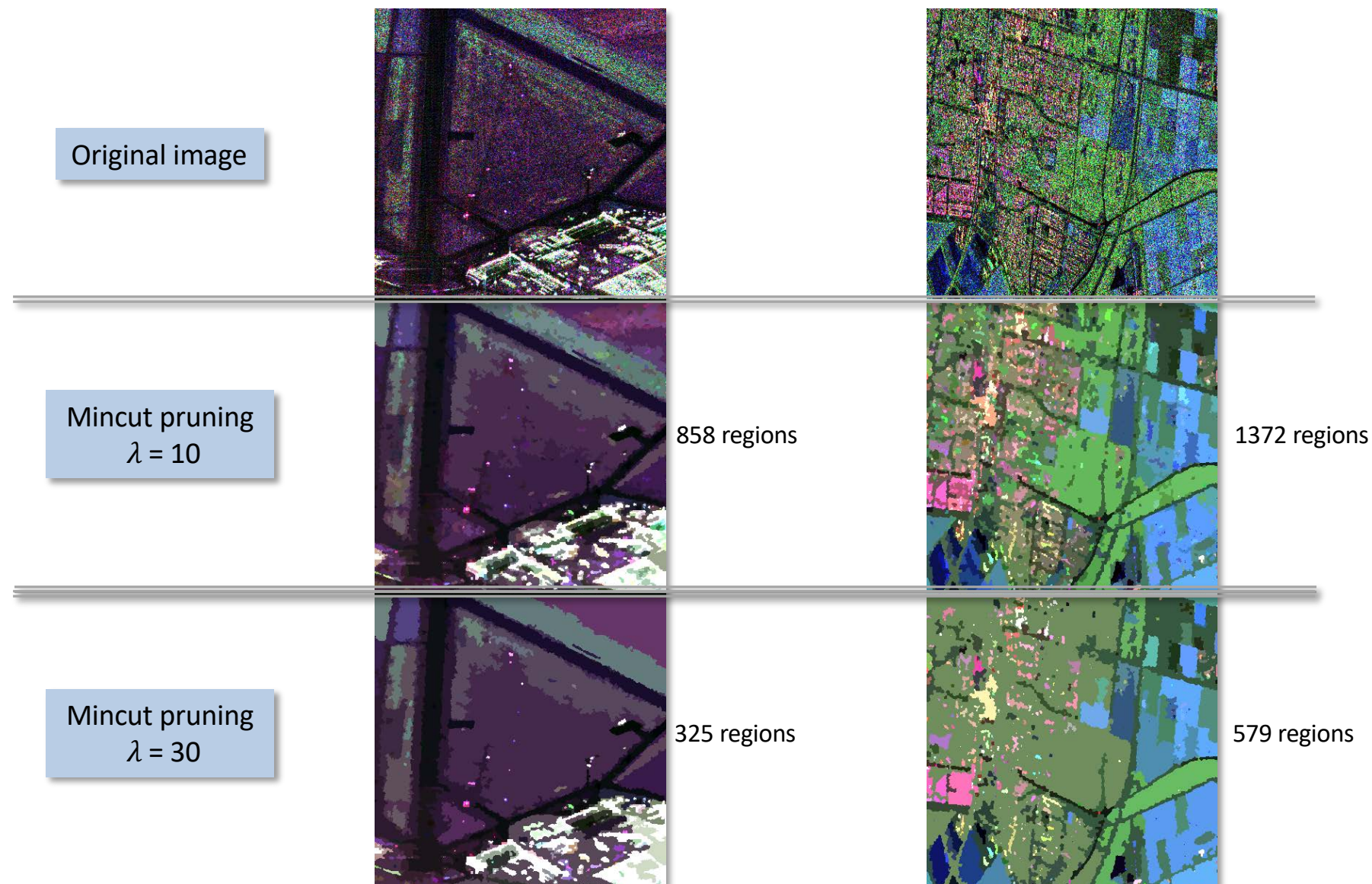
Justify the BPT-based strategy

Results: Synthetic images



Segmentation / Speckle filtering

Result on real images

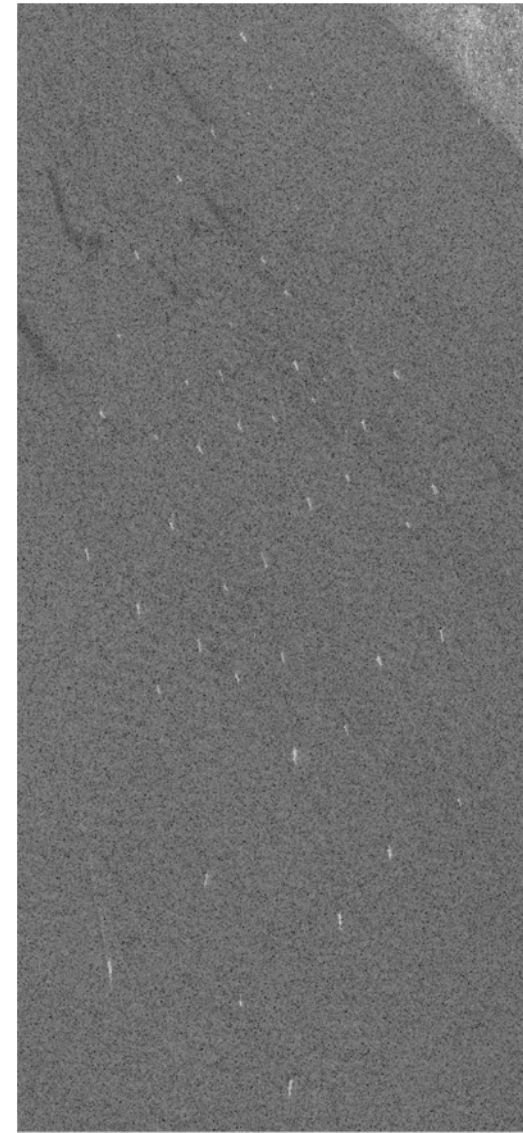


Outline

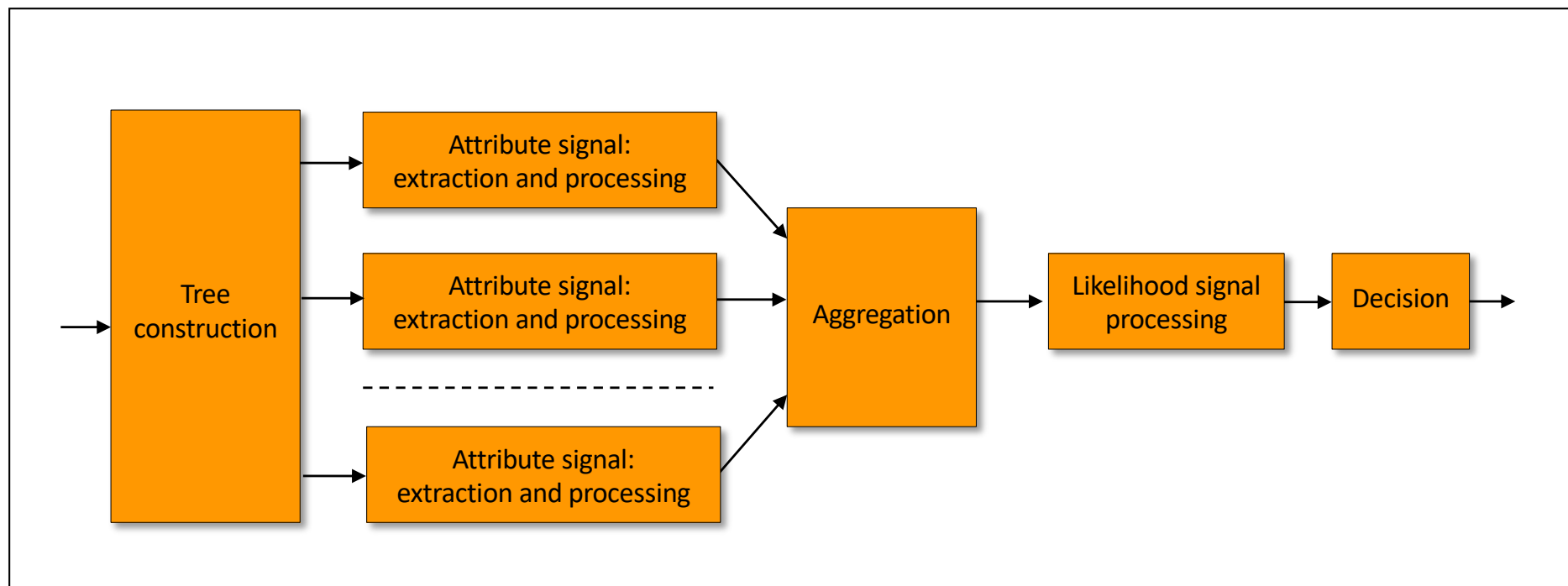
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Graph signal processing

- In the case of Polarimetric image segmentation with BPT, the tree node were populated with some homogeneity attributes and the **attributes values were used to define the optimal pruning.**
- Can attribute values be considered as a signal whose support is the tree itself?
- Lead to **Graph Signal Processing.**
- Illustration: Ship detection in SAR images.
- Intuition: Ships have...
 - a high gray level value (radiometric attribute)
 - an elliptical shape (geometrical attribute)
 - Let us capture these features in the maxtree representation.

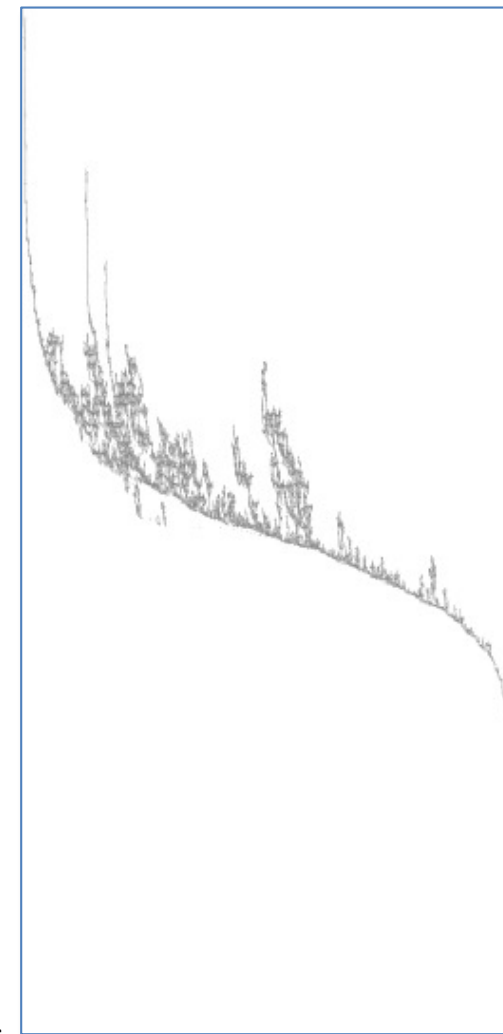
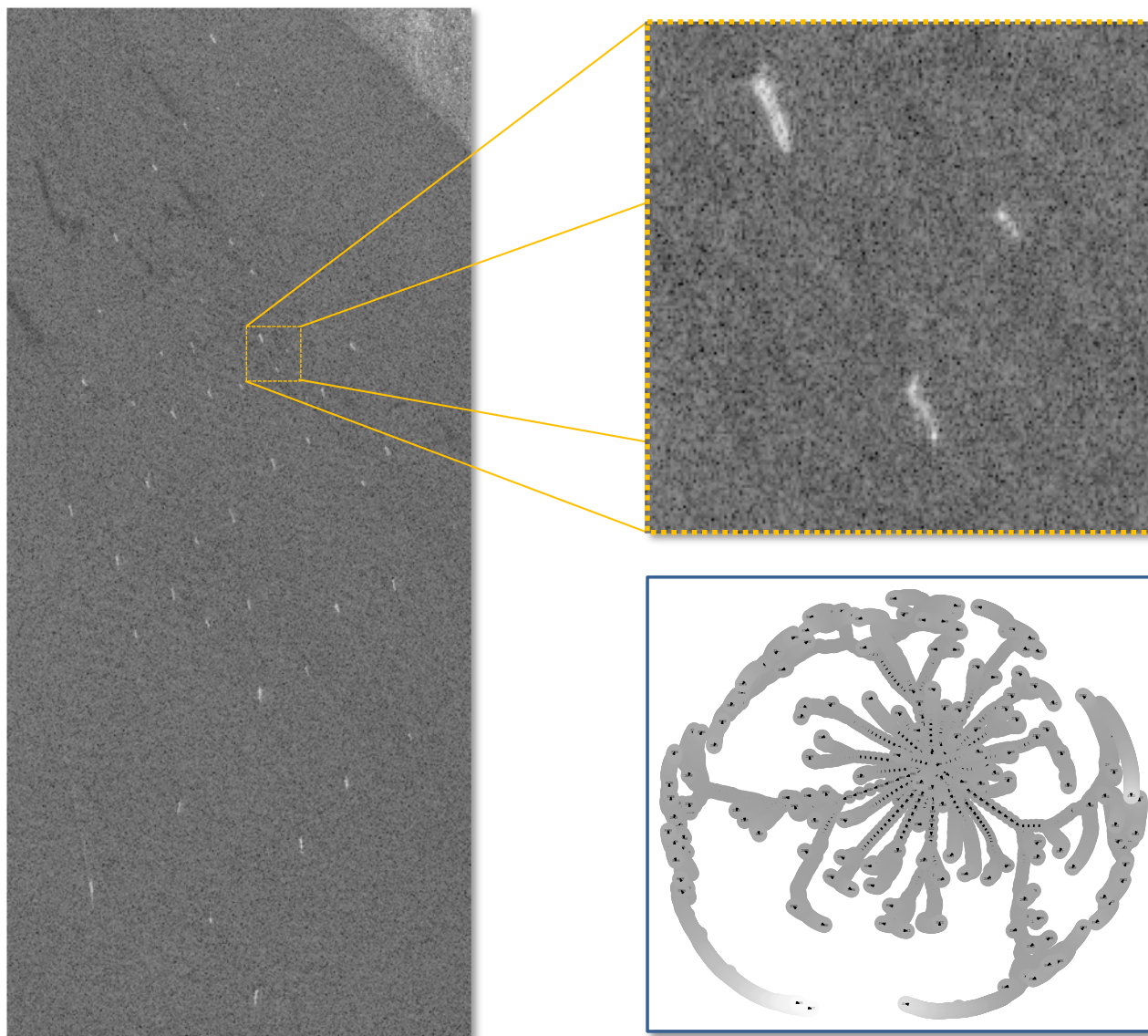


Processing Strategy



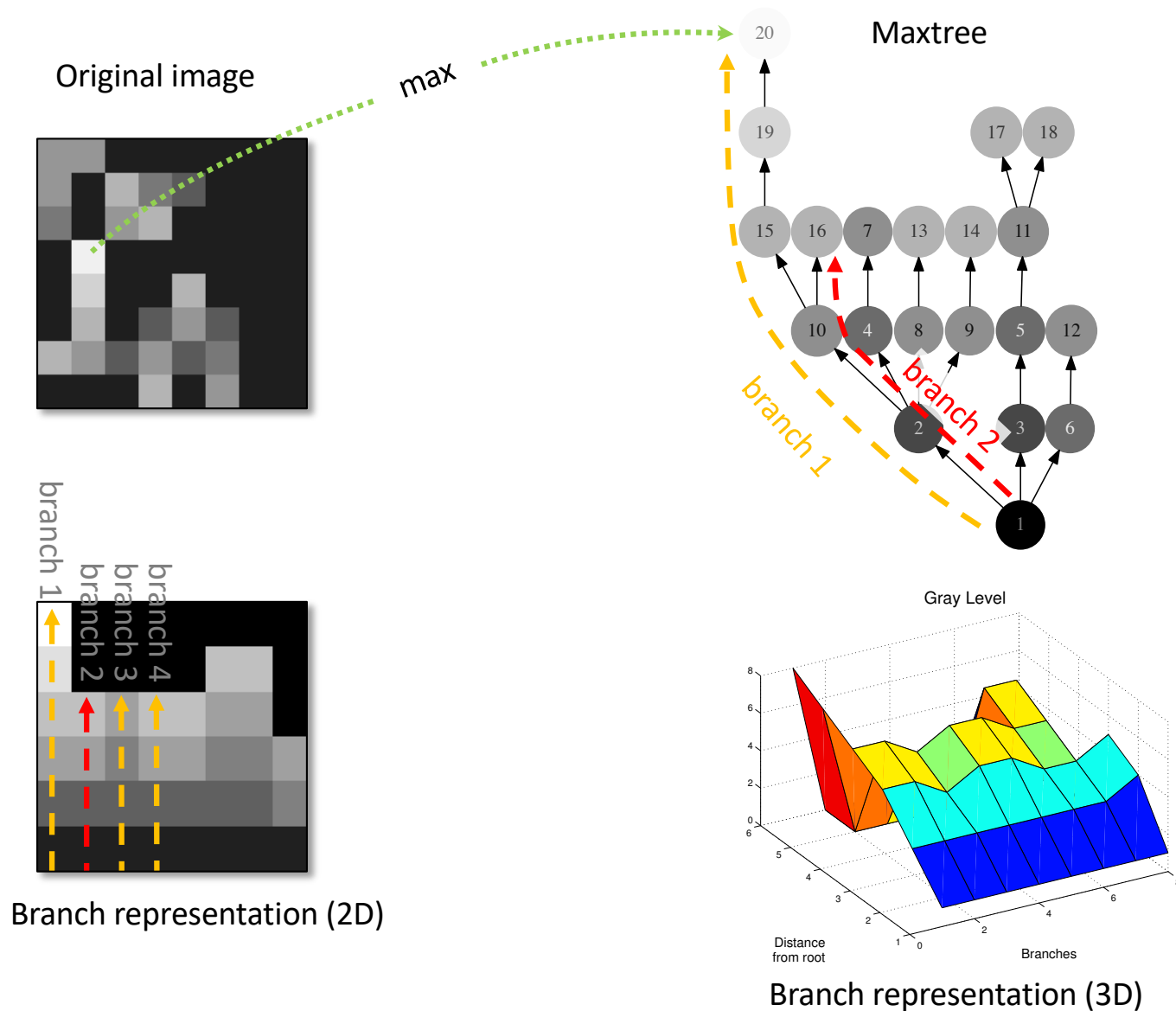
Graph signal processing

Classical tree representations are inappropriate to observe graph signals.

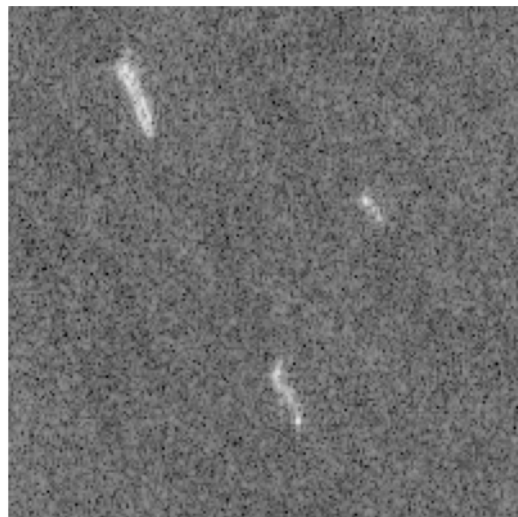


Even sophisticated graph drawing have limited success (graphviz **scalable force directed placement** (sfdp)).

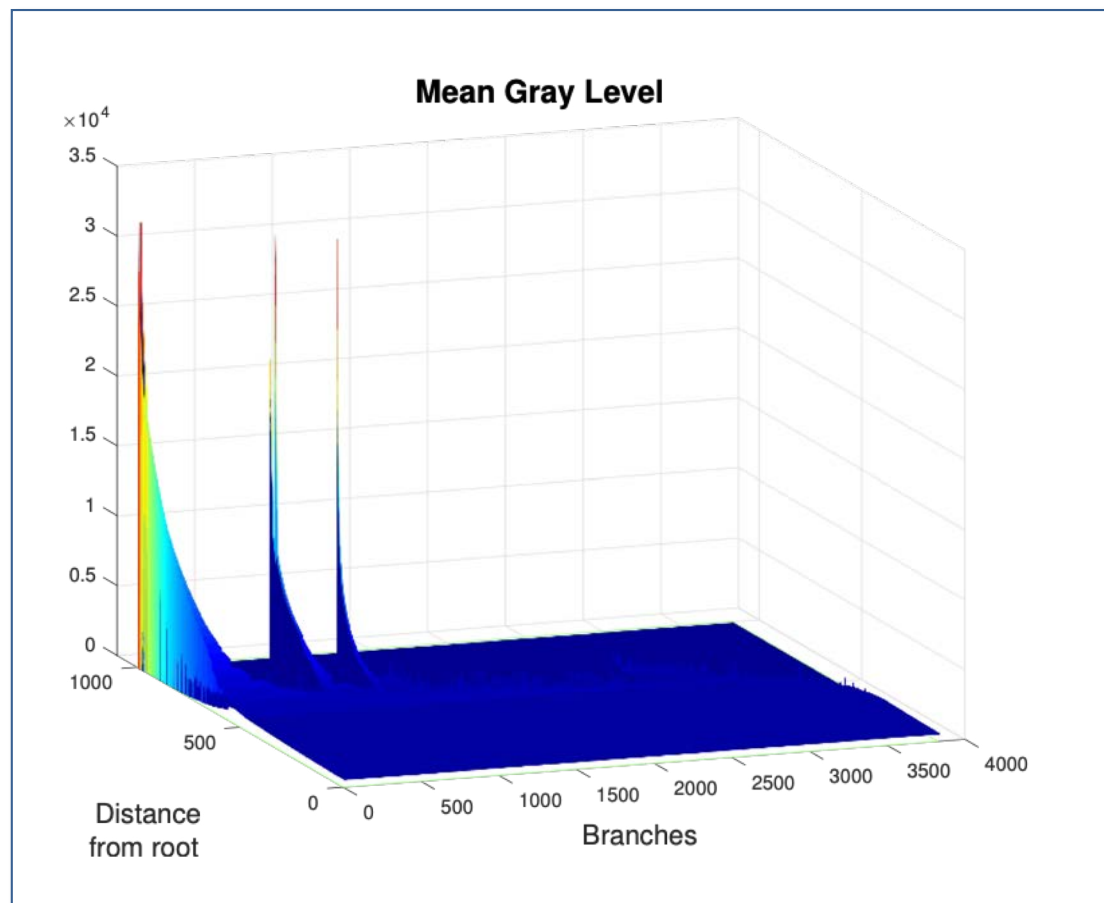
Graph signal processing: Branch representation



Graph signal processing: Attribute



Ships have a high gray level value:
Gray level graph signal

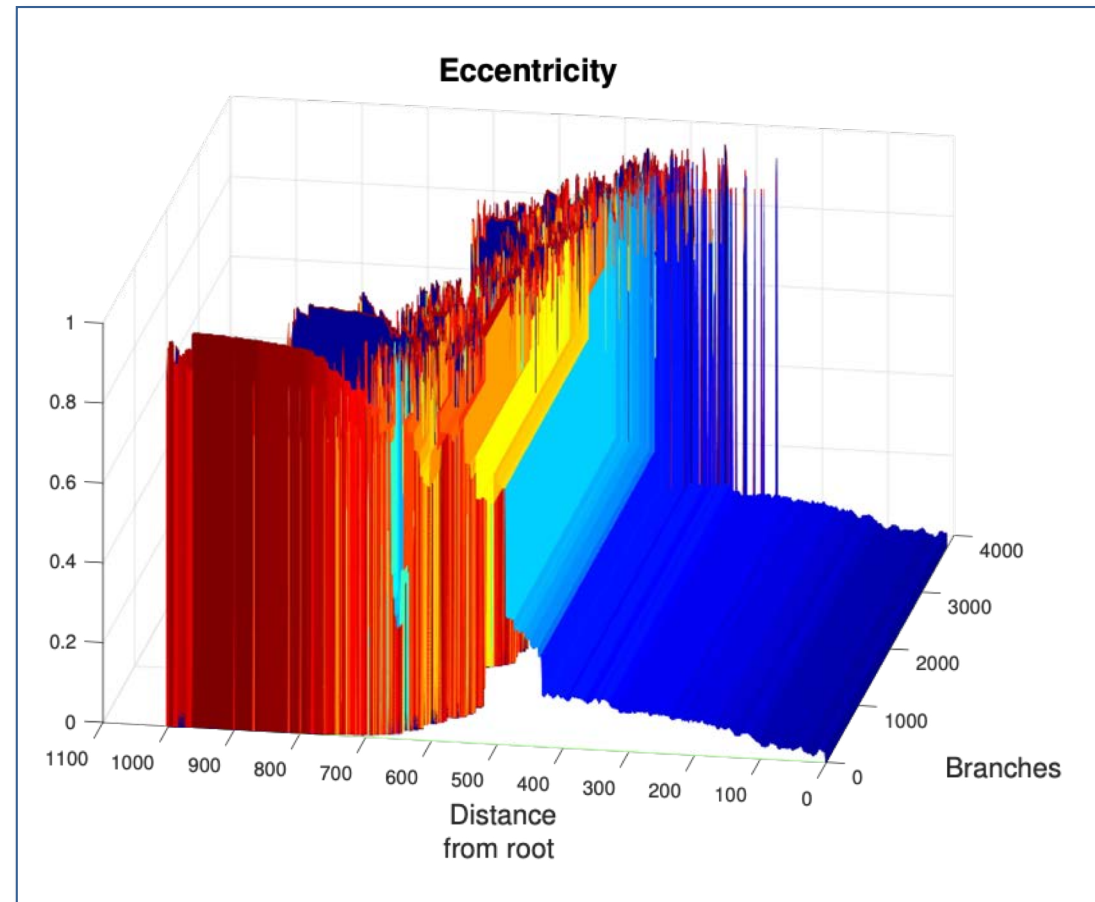
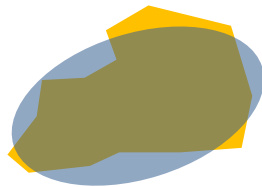


Graph signal processing: Attribute

Ships have a elliptical shape:

Eccentricity graph signal

Attribute: the eccentricity of the ellipse that has the same second-moments as connected component represented by the maxtree node.

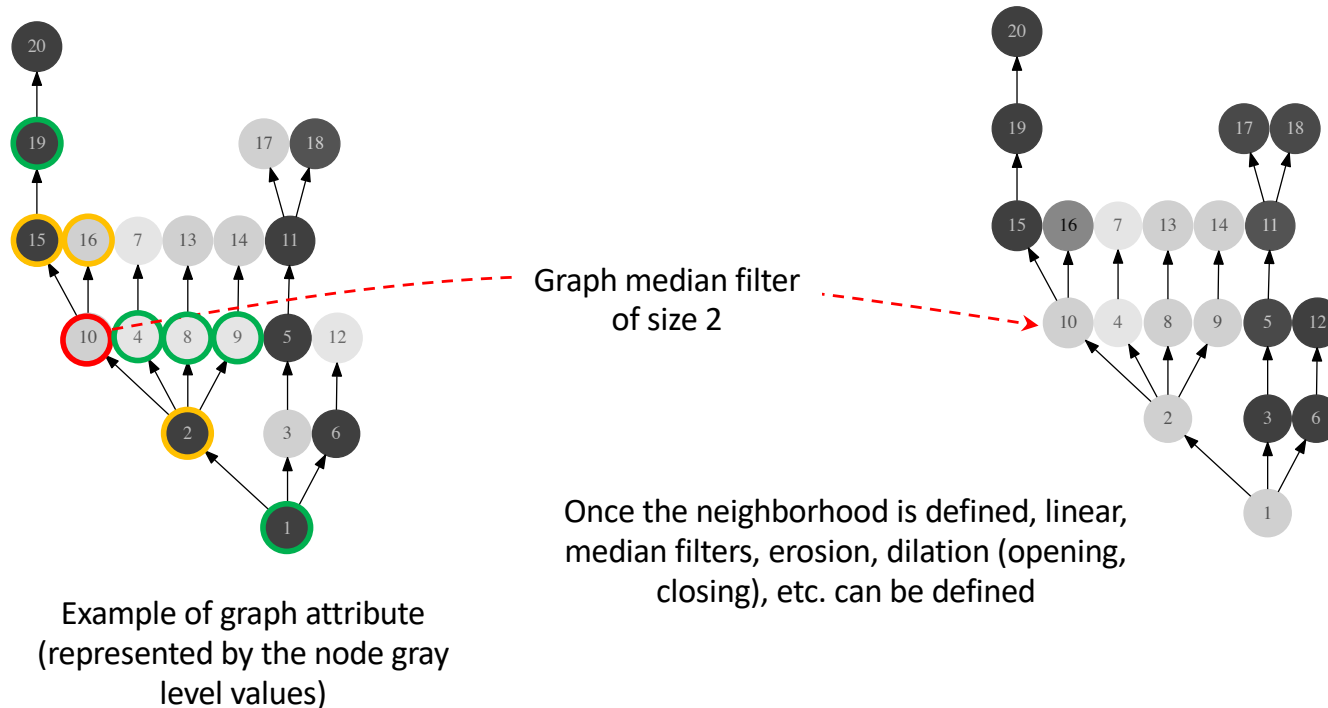


Random fluctuations of the attribute values along the tree branches -> **need of filtering**

Graph signal processing: Filters

Graph filter:

- For each node, define the set of K-hop neighboring nodes and
- Apply a classical filter on the set of nodes

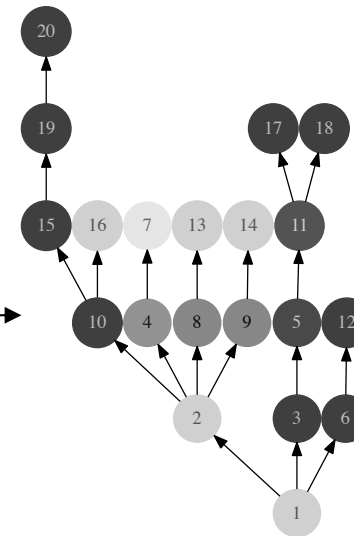
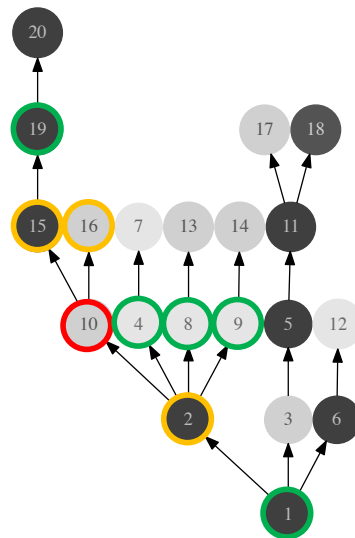


Tree are particular graphs but the connectivity towards ancestors may be different from the one towards descendant.

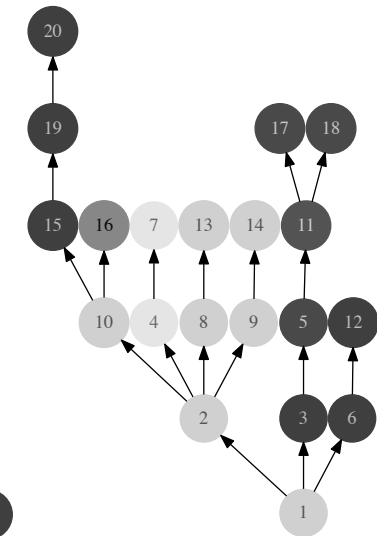
Graph signal processing: Filters

Tree filter:

- The neighborhood of a node is exclusively composed of **all its descendants** and **all its ancestors** that are at distance lower or equal to a given value (not the descendants of the ancestors).



Tree median filter
of size 2

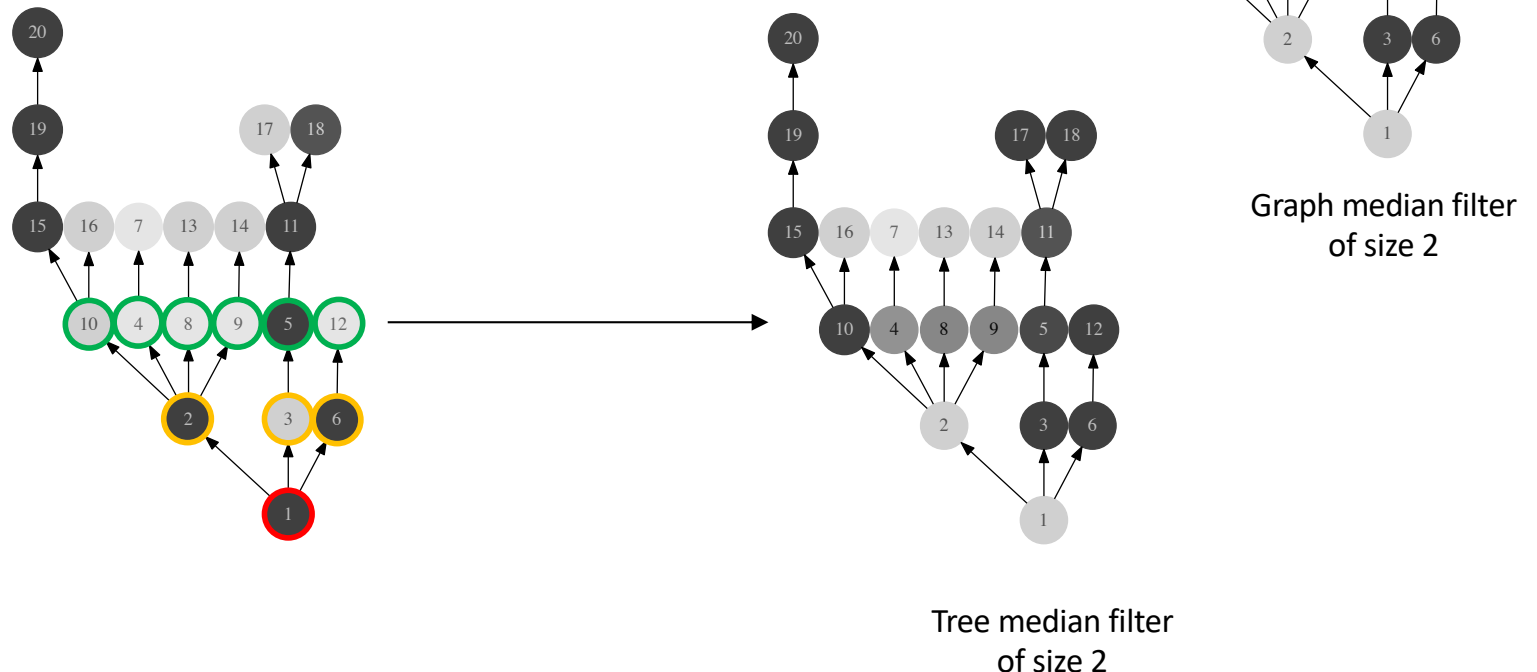


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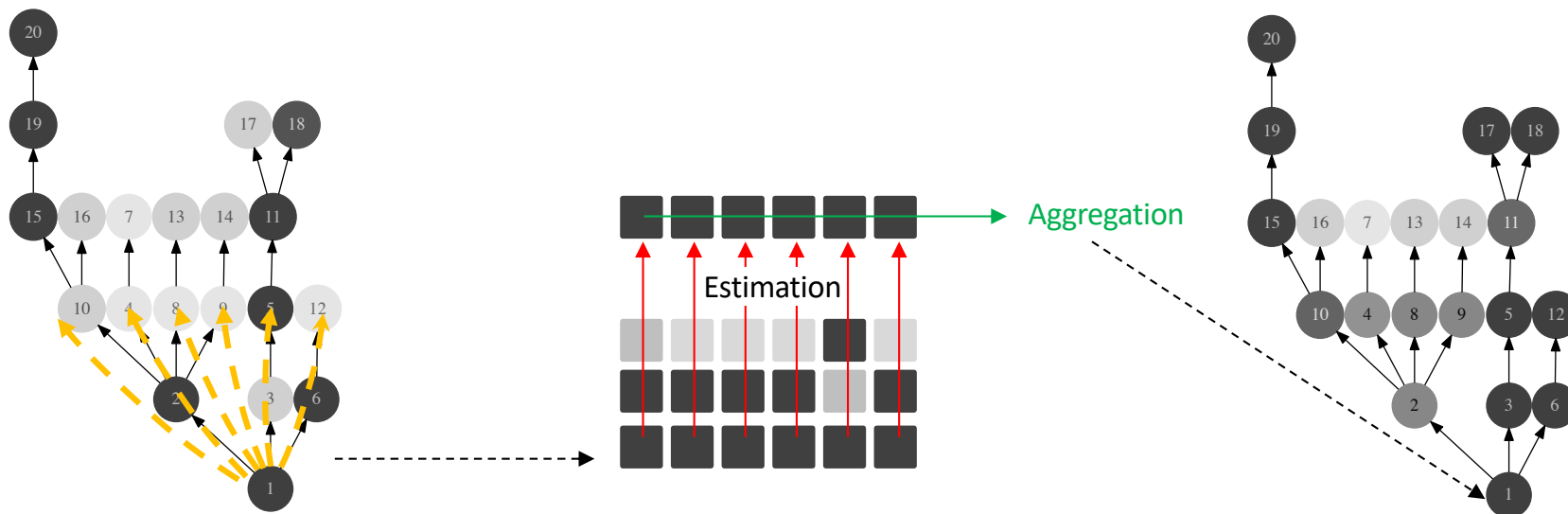


The disparity of nodes may lead to unintuitive filtering results (ex: the value node 1 is largely dominated by node 2 and its numerous descendants).

Graph signal processing: Filters

Branch filter:

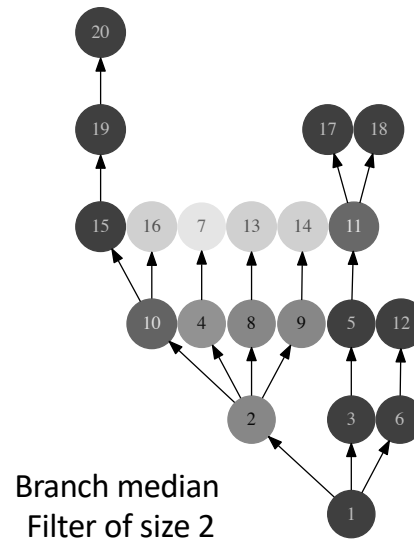
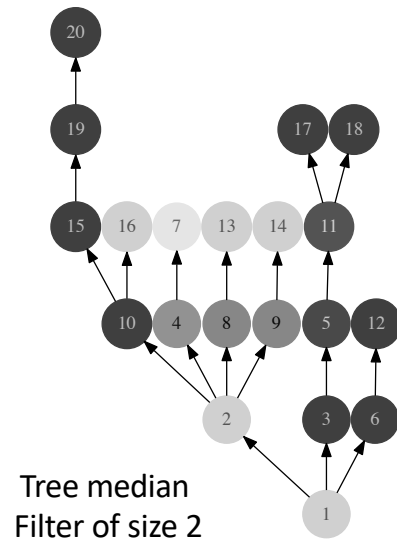
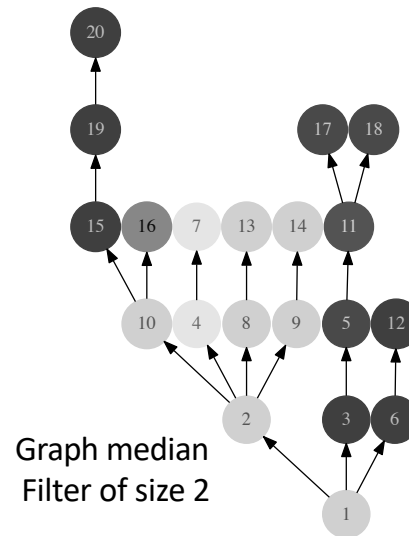
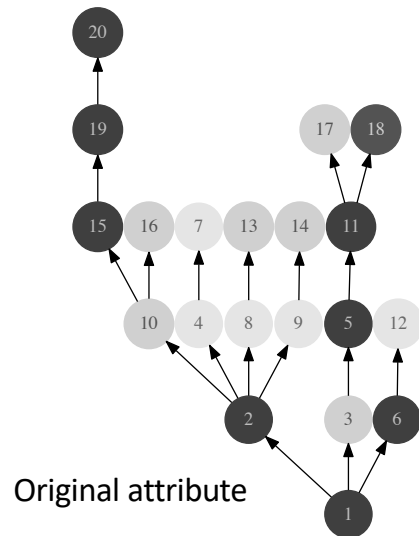
- Two step approach: Estimation and aggregation.
- Estimation: Collect the attribute values on each branch and apply a filter.
- Aggregation: Filter the values obtained for each branch and define the final value.



Filter	Estimation	Aggregation
Branch Mean	1D Mean	Mean
Branch Median	1D Median	Median
Branch Erosion	1D Erosion	Min
Branch Dilation	1D Dilation	Max

TABLE I
DEFINITION OF THE ESTIMATION AND THE AGGREGATION STEPS FOR
ELEMENTARY BRANCH FILTERS

Graph signal processing: Filters

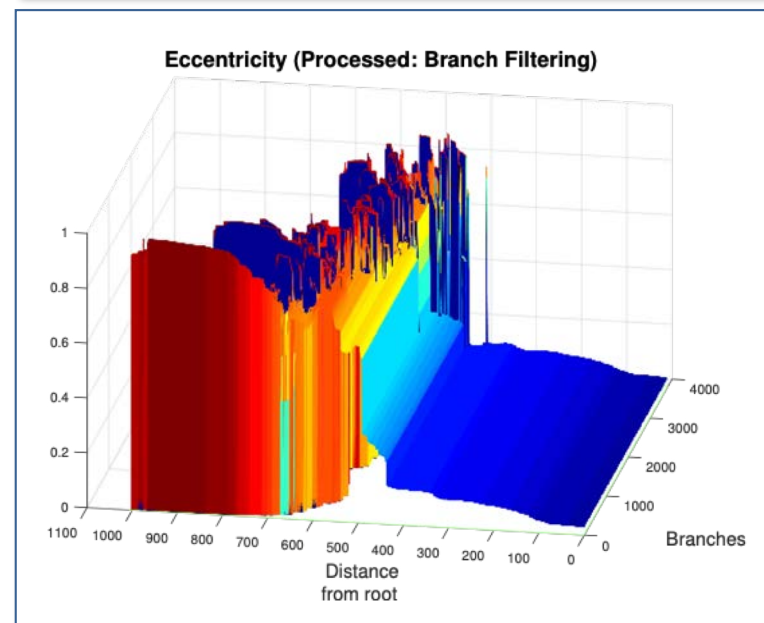
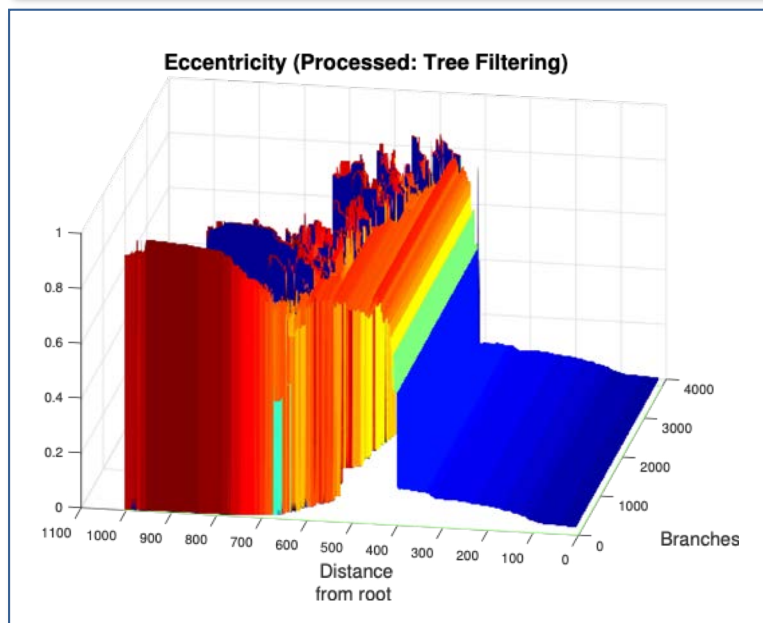
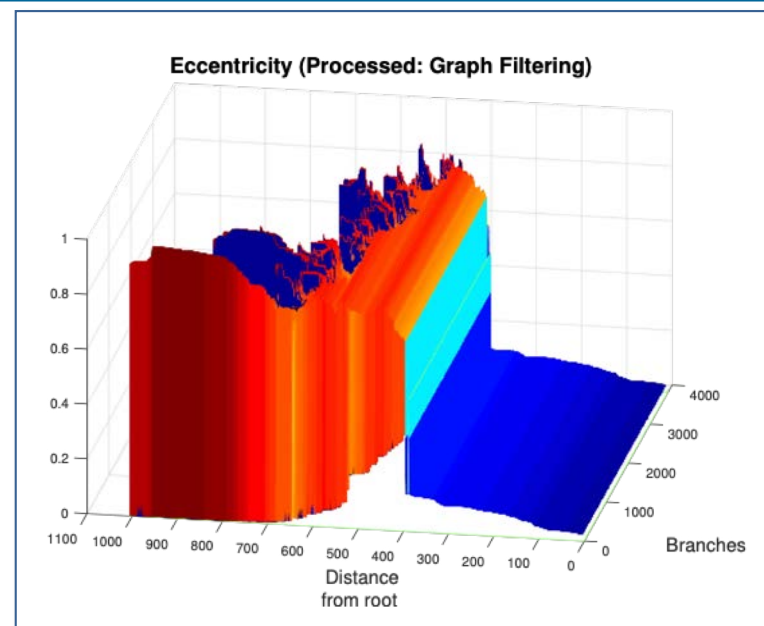
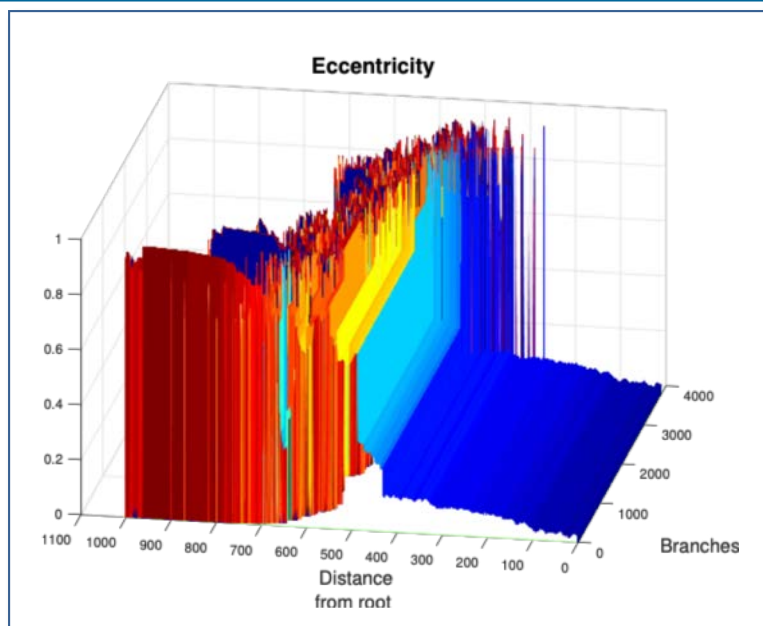


In some cases the distinction disappears:

- Graph and Tree filters of size one are equivalent.
- Flat Branch and Tree erosion and dilation (opening and closing) are equivalent.

Filter	Estimation	Aggregation
Branch Mean	1D Mean	Mean
Branch Median	1D Median	Median
Branch Erosion	1D Erosion	Min
Branch Dilation	1D Dilation	Max

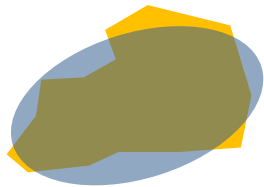
Graph signal processing: Filters



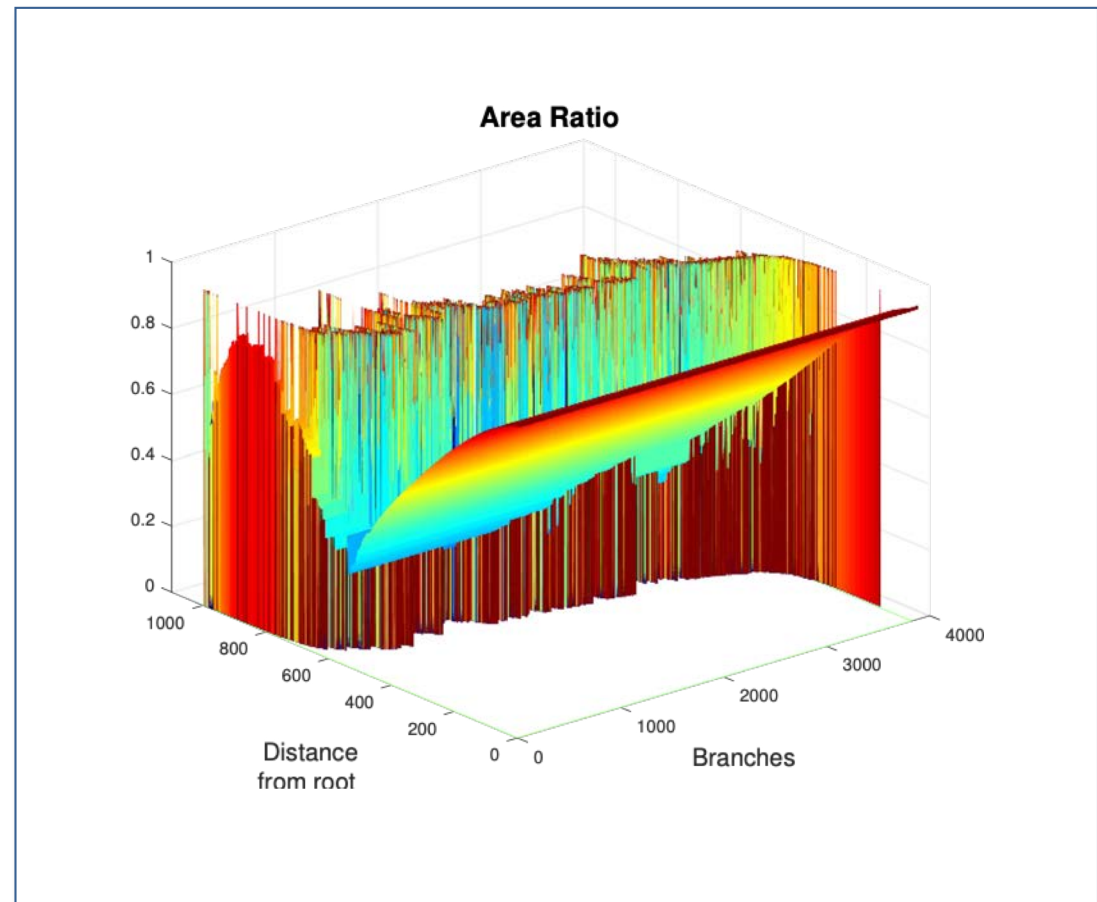
Graph signal processing: Filters

Ships have a elliptical shape:
Eccentricity graph signal

Attribute: Area ratio

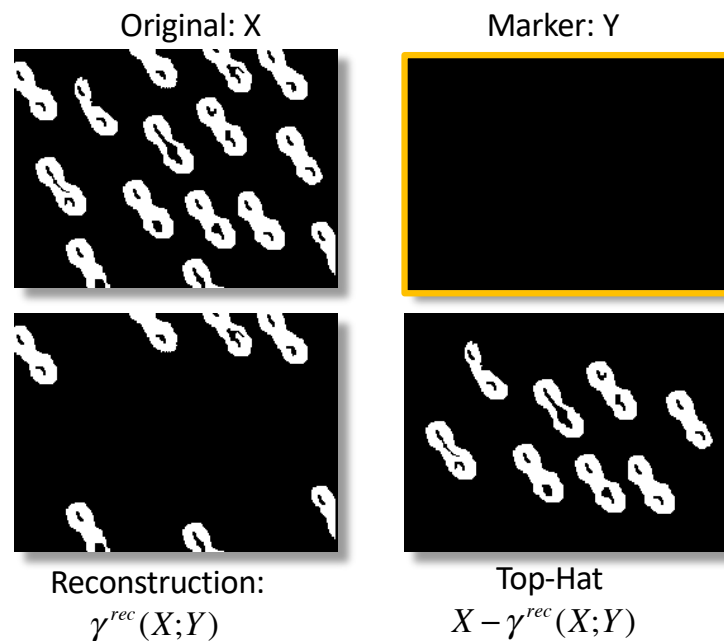


Relation between the area of
the connected component and
that of the best fitting ellipse



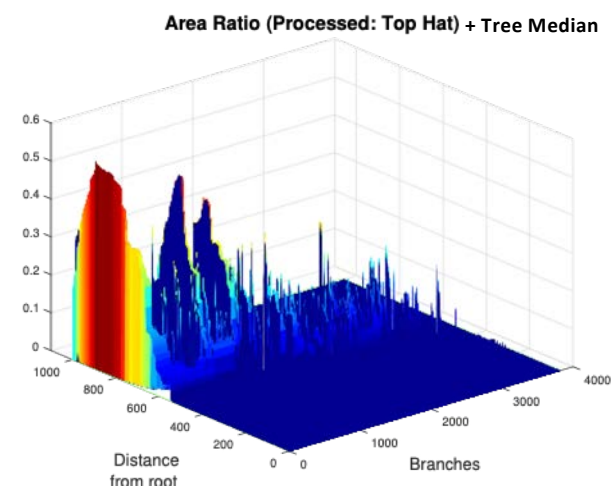
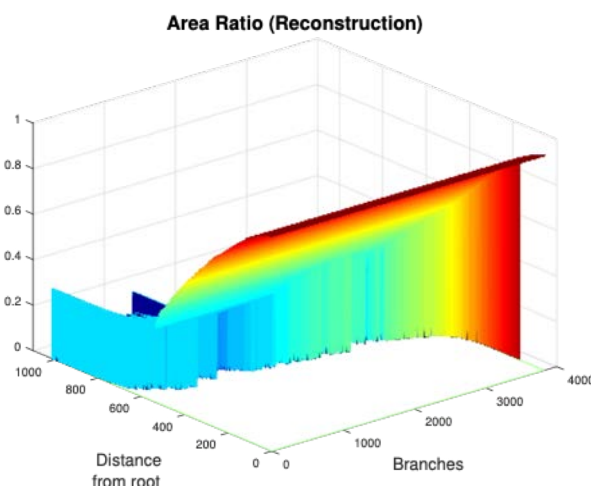
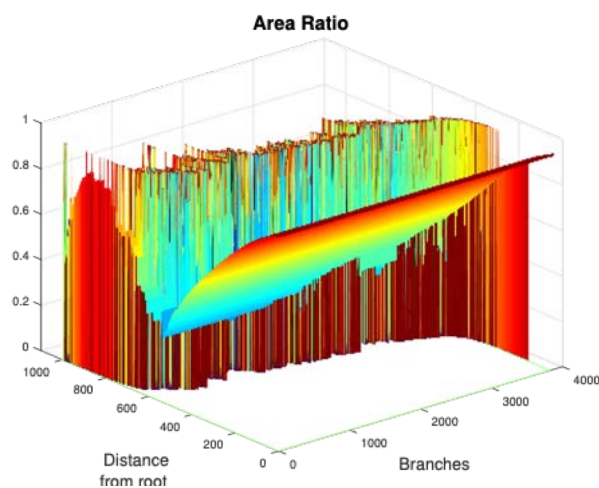
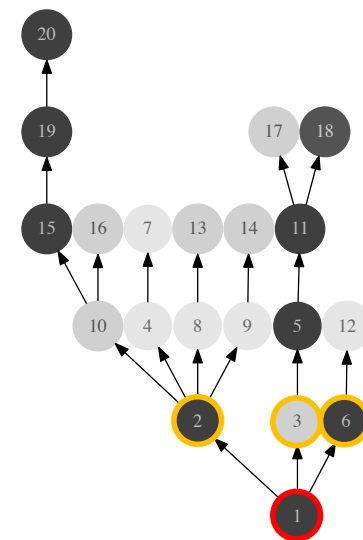
High values on nodes corresponding to ships but also for the root of the tree (image support can be rather well approximated by an ellipse) -> **need of a cleaning process similar to removing maxima that intercept the image border.**

Graph signal processing: Filters

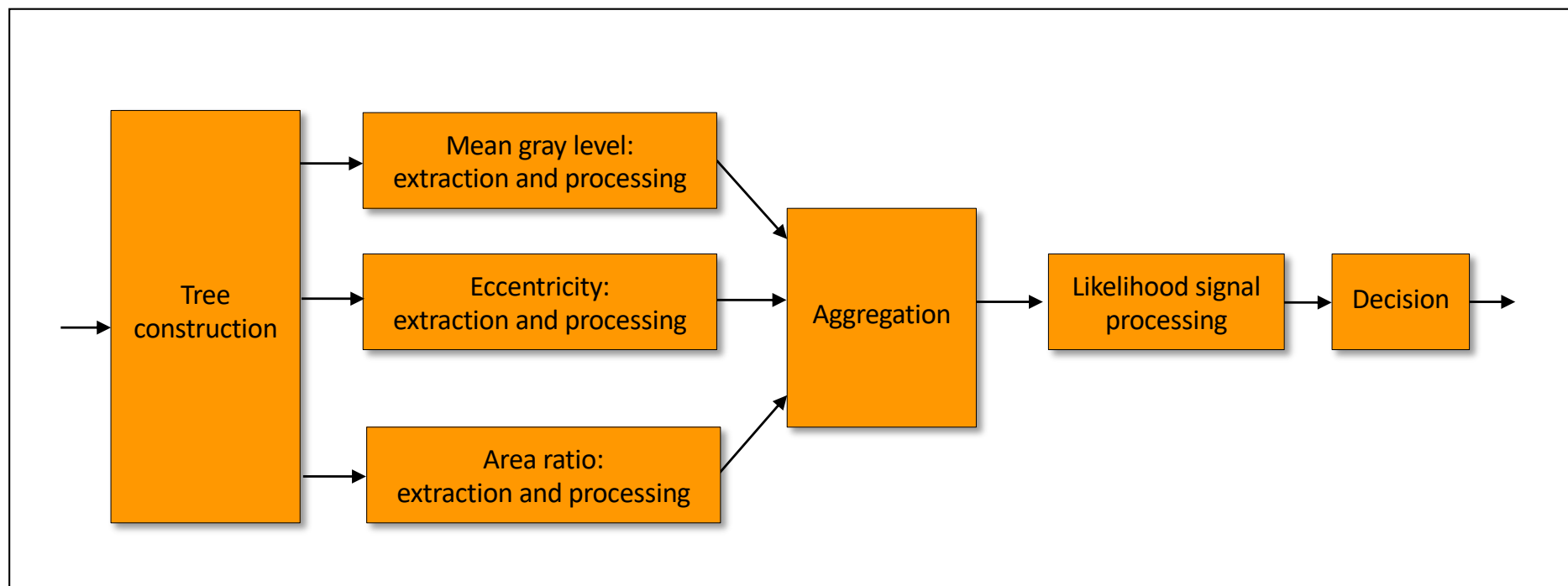


$$\gamma^{rec}(X; Y) = \lim_{k \rightarrow \infty} Y_k$$

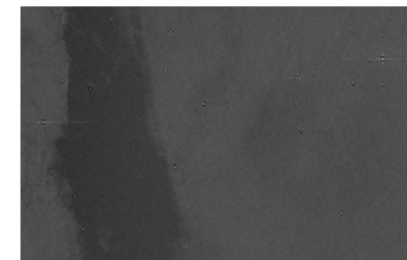
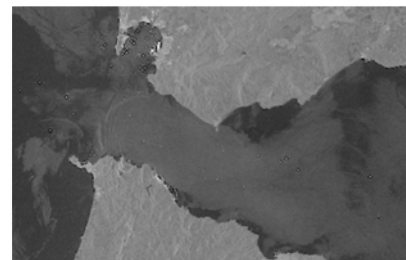
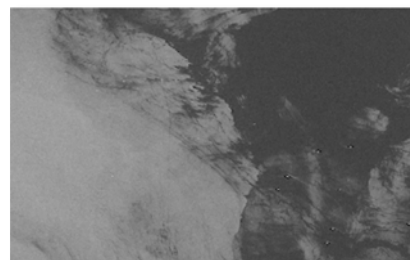
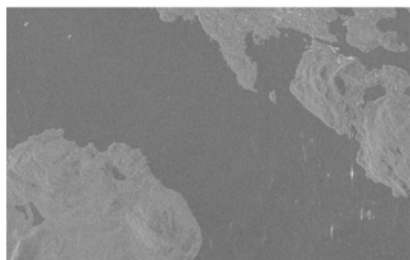
$$Y_k = \delta_S(Y_k) \wedge X$$



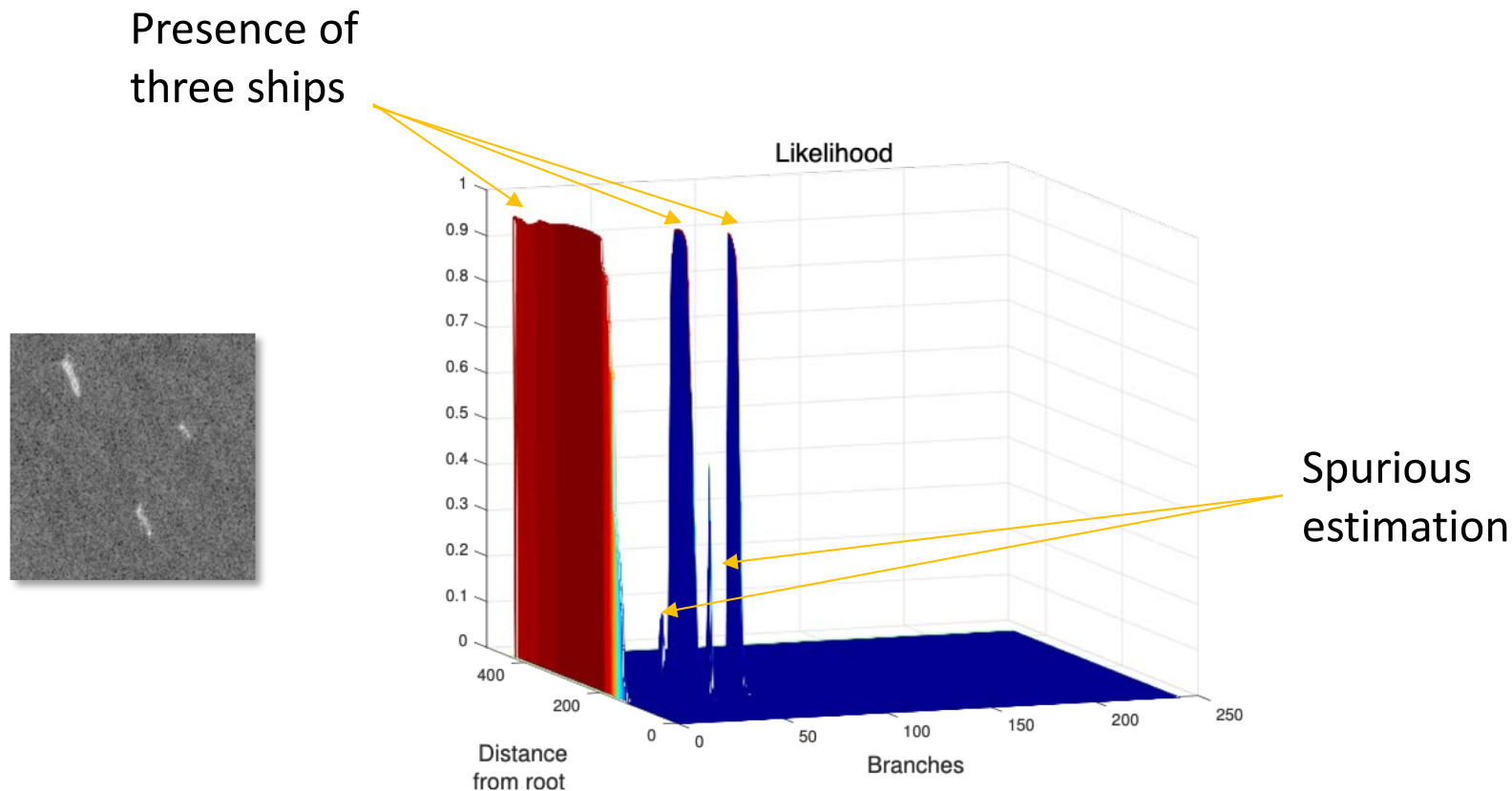
Processing Strategy



Aggregation performed by a SVM (estimates the likelihood of a node to represent a ship)



Likelihood estimation with SVM

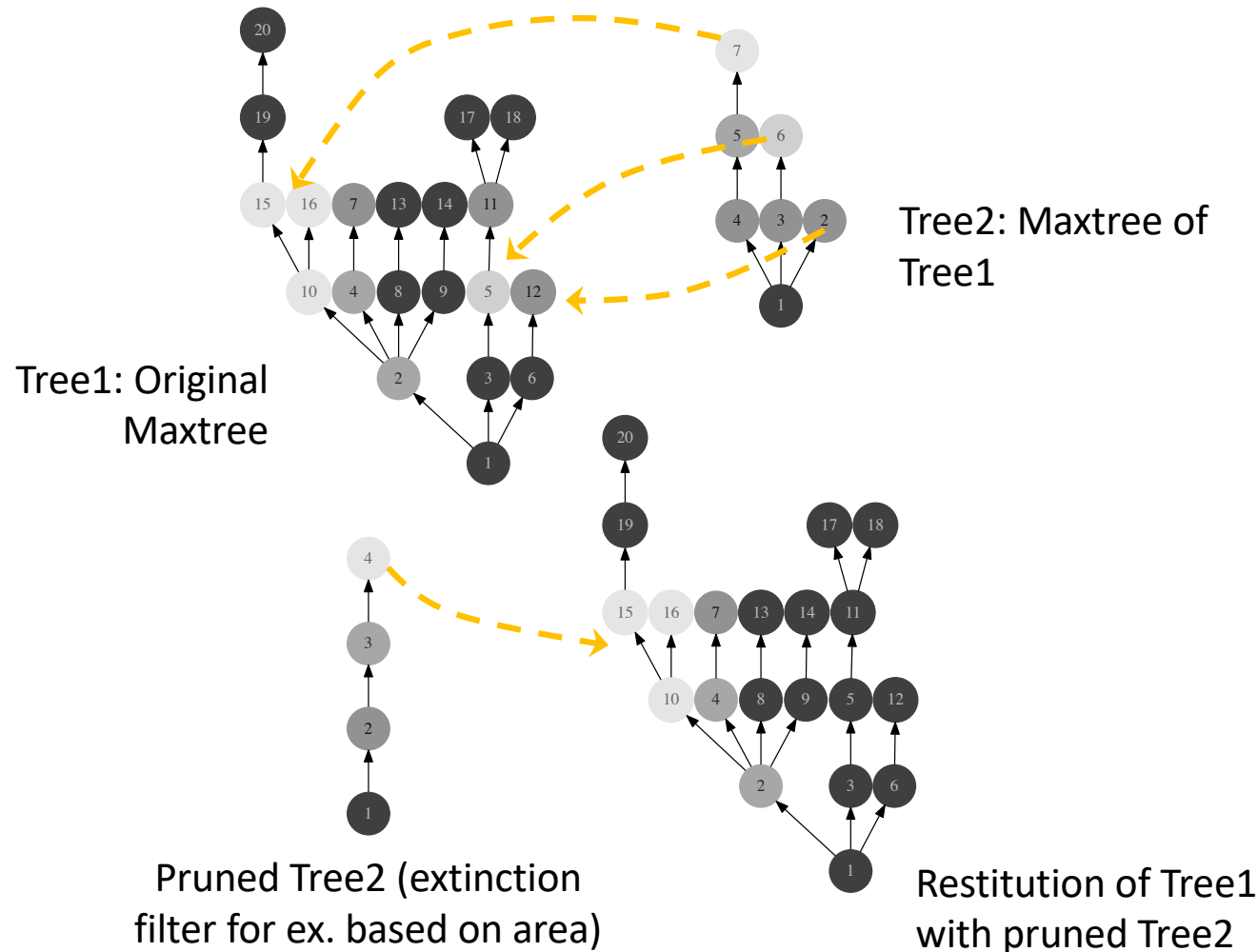


Aggregation performed by a SVM (estimates the likelihood of a node to represent a ship)

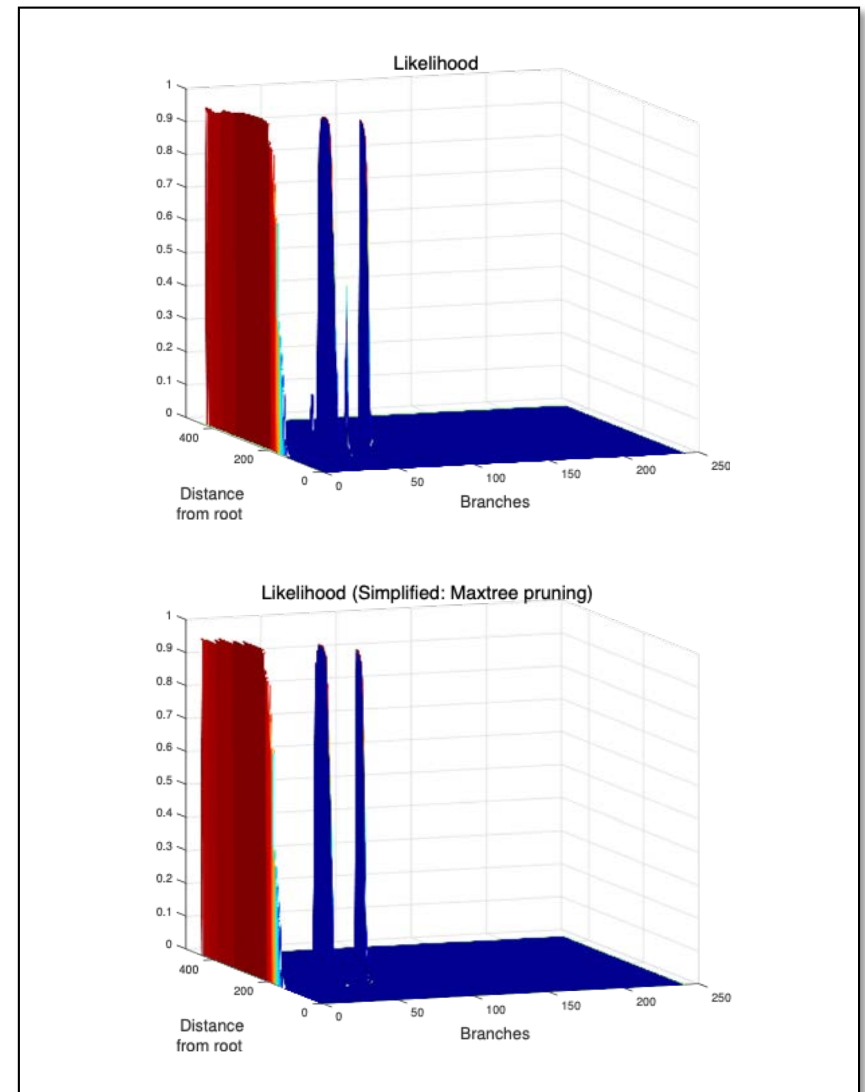
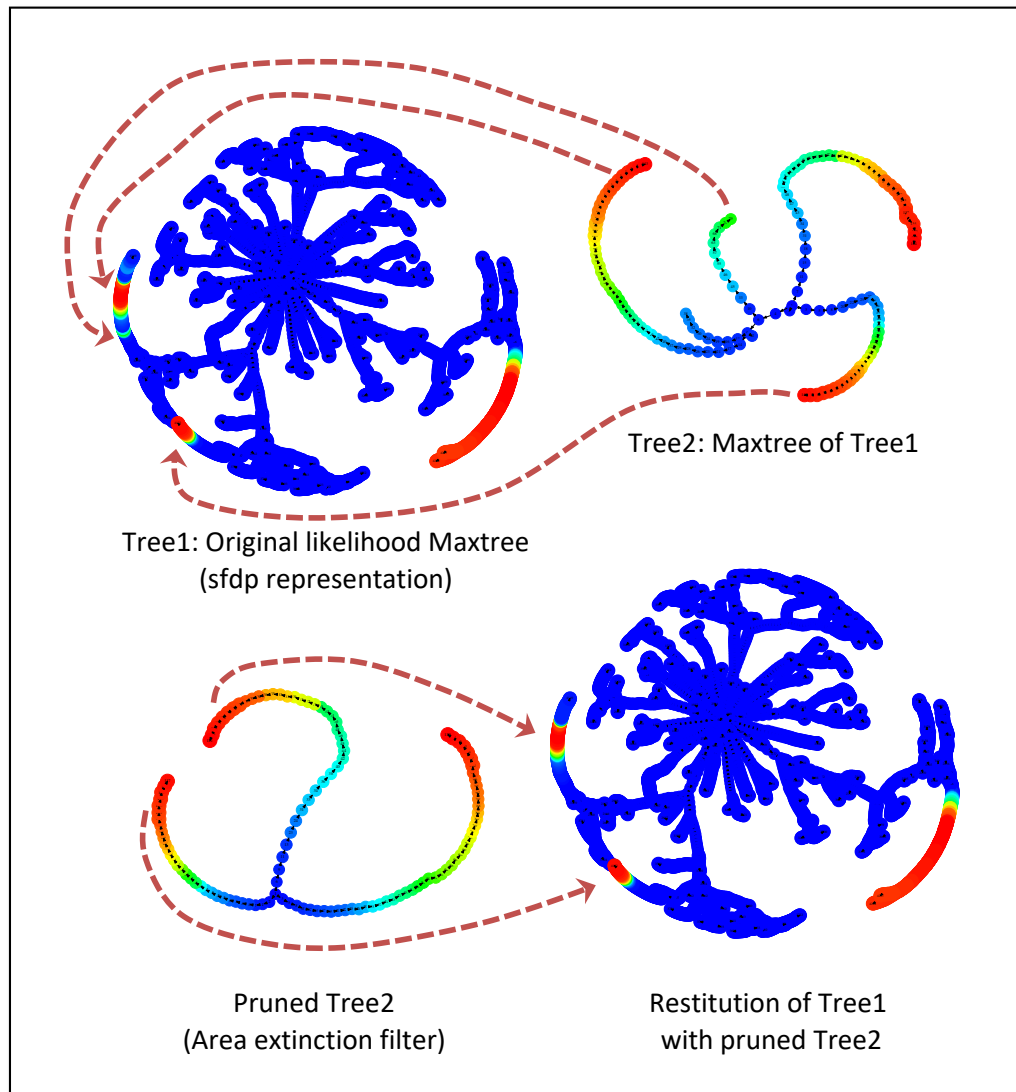
Likelihood signal processing

Representing and processing the attribute maxima with a maxtree.

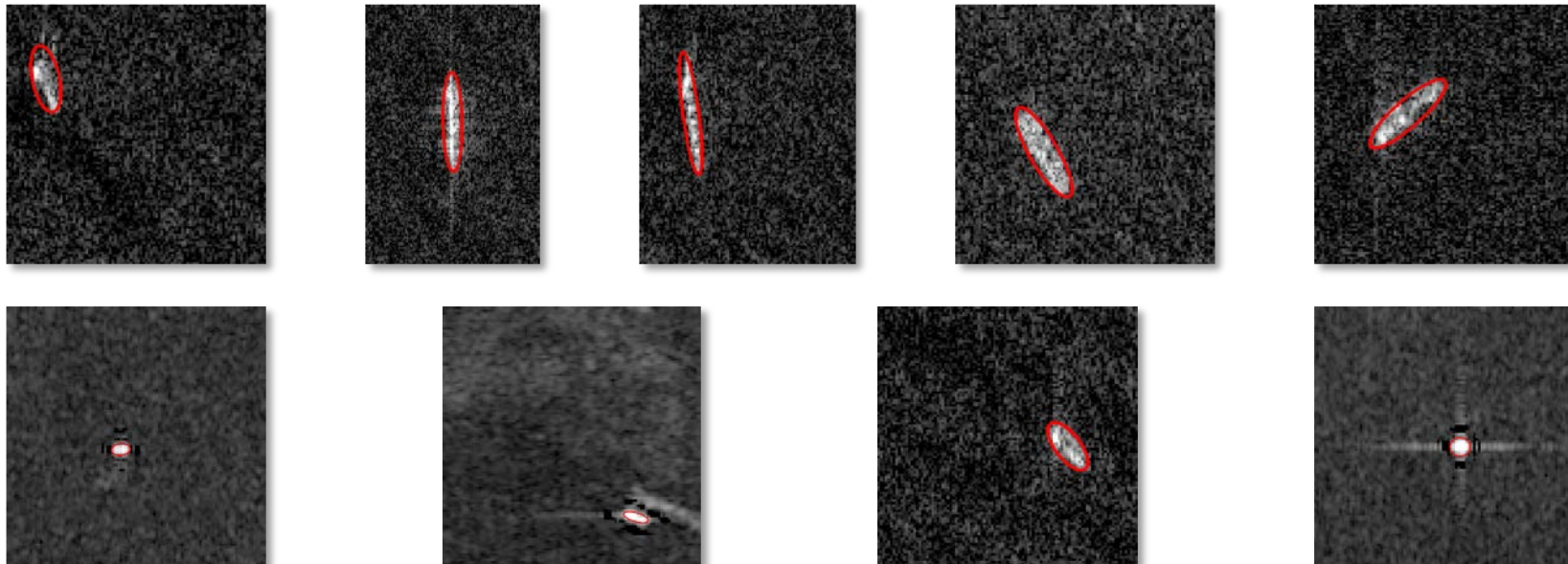
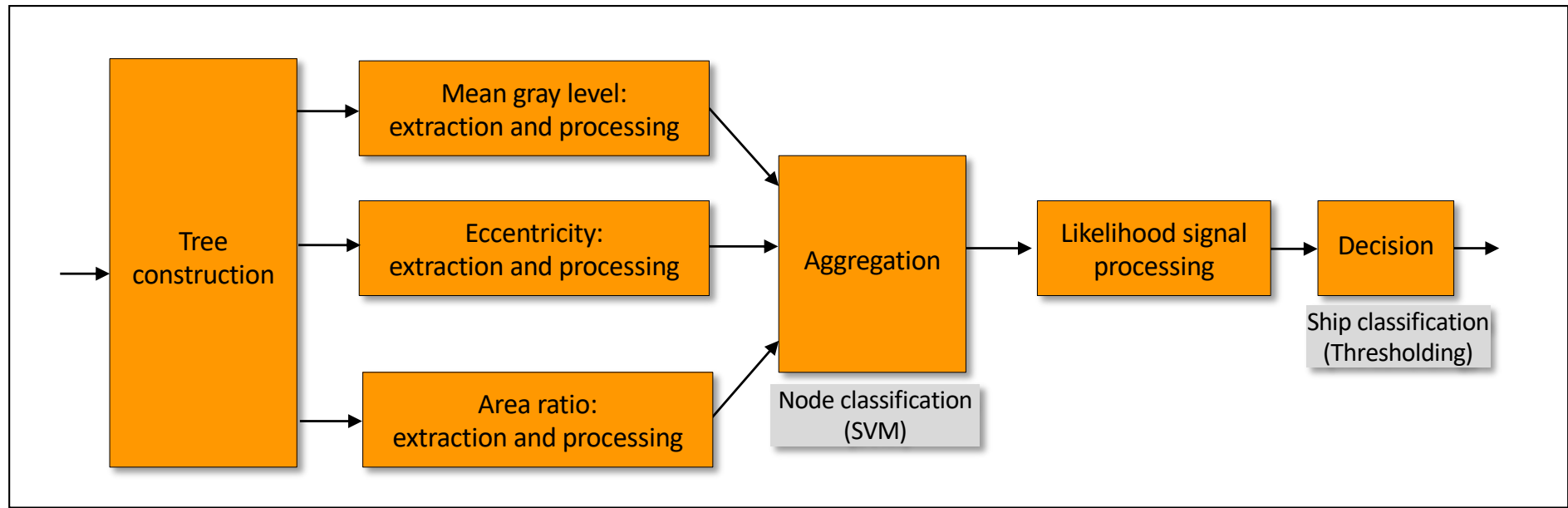
Y. Xu, T. Géraud and L. Najman, Connected Filtering on Tree-Based Shape-Spaces, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 38(6), 2016.



Likelihood signal processing



Results



Results: node classification

Set	Precision	Recall	F-Score
Training	0.853	0.875	0.864
Validation	0.840	0.868	0.853
Test	0.828	0.892	0.858

No overtraining

Node classification results **without** attribute filtering

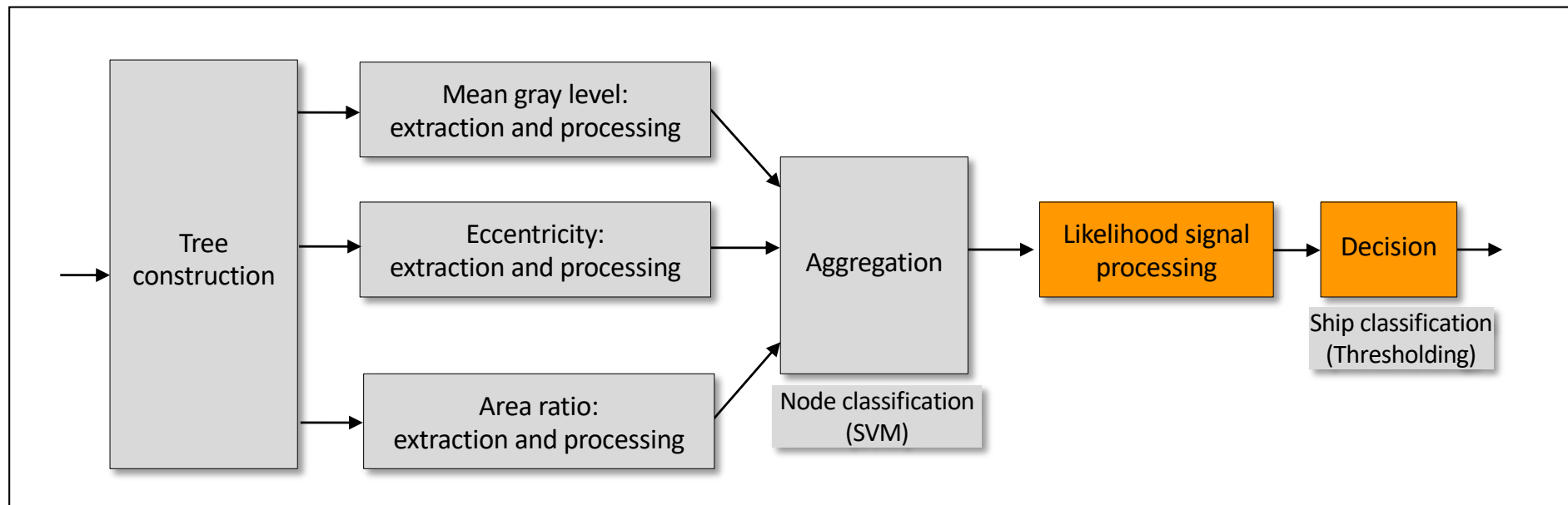
Graph filters				Tree filters			Branch filters		
Mean	Precision	Recall	F-Score	Precision	Recall	F-Score	Precision	Recall	F-Score
Training	0.918	0.861	0.889	0.887	0.845	0.865	0.927	0.922	0.925
Validation	0.915	0.866	0.890	0.867	0.840	0.853	0.912	0.924	0.918
Test	0.913	0.874	0.893	0.866	0.843	0.854	0.926	0.934	0.930
Median	Precision	Recall	F-Score	Precision	Recall	F-Score	Precision	Recall	F-Score
Training	0.884	0.872	0.878	0.879	0.906	0.892	0.897	0.919	0.908
Validation	0.863	0.869	0.866	0.875	0.905	0.889	0.897	0.931	0.914
Test	0.887	0.890	0.888	0.876	0.910	0.893	0.891	0.930	0.910
Opening	Precision	Recall	F-Score	Precision	Recall	F-Score	Precision	Recall	F-Score
Training	0.905	0.881	0.893	0.924	0.943	0.933	0.924	0.943	0.933
Validation	0.906	0.874	0.890	0.922	0.946	0.934	0.922	0.956	0.934
Test	0.886	0.871	0.878	0.920	0.953	0.936	0.920	0.953	0.936
Closing	Precision	Recall	F-Score	Precision	Recall	F-Score	Precision	Recall	F-Score
Training	0.844	0.861	0.872	0.897	0.908	0.902	0.897	0.908	0.902
Validation	0.887	0.871	0.879	0.890	0.927	0.908	0.890	0.927	0.908
Test	0.867	0.866	0.866	0.882	0.928	0.905	0.882	0.928	0.905

Node classification results **with** attribute filtering

Shows the interest of attribute graph signal processing and of going beyond classical graph filters.

Results: Ship classification

Ship detection approach	Precision	Recall	F-Score
Proposed approach without area extinction filter	0.878	1.000	0.935
Proposed approach with area extinction filter	0.947	1.000	0.973



Shows the interest of the area extinction filter processing the likelihood information.

Results: Ship classification

Ship detection approach	Precision	Recall	F-Score
Proposed approach without area extinction filter	0.878	1.000	0.935
Proposed approach with area extinction filter	0.947	1.000	0.973
CFAR approach [1]	0.865	0.889	0.877
Wavelet-based approach [2]	0.857	0.923	0.889
GLRT approach [3]	0.872	0.944	0.907
Entropy-based dissimilarity [4]	0.800	1.000	0.889

- [1] Y. Cui, J. Yang, , and Y. Yamaguchi, “CFAR ship detection in SAR images based on lognormal mixture models,” in 3rd IEEE Int. Asia- Pacific Conf. on Synthetic Aperture Radar, IEEE, Ed., Seoul, South Korea, 2011.
- [2] M. Tello, C. López-Martínez, and J. Mallorqui, “A novel algorithm for ship detection in SAR imagery based on the wavelet transform,” IEEE Geoscience and Remote Sensing Letters, vol. 2, no. 2, pp. 201–205, 2005.
- [3] A. Marino, M. J. Sanjuan-Ferrer, I. Hajnsek, and K. Ouchi, “Ship detection with spectral analysis of synthetic aperture radar: A comparison of new and well-known algorithms,” Remote Sensing, vol. 7, no. 5, pp. 5416–5439, 2015.
- [4] X. Wang and C. Chen, “Ship detection for complex background SAR images based on a multiscale variance weighted image entropy method,” IEEE Geoscience and Remote Sensing Letters, vol. 14, no. 2, pp. 184– 187, 2017.

Conclusions

- **Show the interest of processing images from a structured representation of its pixels:**
 - Hierarchical representation leading to trees: Max/Mintree and BPT as examples
- **Processing strategies:**
 - Act on the support of the tree: Pruning
 - Useful for segmentation and for filtering (connected operators)
 - Populate the tree with attributes values and treat these attribute values as graph signals
 - Filtering tools: Graph, Tree and Branch versions
 - Morphological reconstruction
 - Connected filters on the graph signal (either by reconstruction or by tree representation).
 - Illustrate the interest for object detection

Thank you for your attention

More information:

- ❑ P. Salembier, Liesegang, S., and López-Martínez, C., “Ship Detection in SAR Images Based on Maxtree Representation and Graph Signal Processing”, IEEE Transactions on Geoscience and Remote Sensing, 2018.
- ❑ P. Salembier and Foucher, S., “Optimum Graph-Cuts for Pruning Binary Partition Trees of Polarimetric SAR images”, IEEE Transactions on Geoscience and Remote Sensing, vol. 54, no. 9, pp. 5493 – 5502, 2016.
- ❑ Matlab toolbox: <https://github.com/imatge-upc/Maxtree-Processing-Toolbox>