CONNECTED OPERATORS BASED ON RECONSTRUCTION PROCESS FOR SIZE AND MOTION SIMPLIFICATION

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ABSTRACT

This paper deals with connected operators based on reconstruction process. Connected operators are filtering tools that act on the connected components of the space where the image is constant, the so-called "flat zones". Intuitively, connected operators can remove boundaries between flat zones but cannot add new boundaries nor shift existing contours. After reviewing the filtering strategy, new operators are proposed to perform size and motion simplification. In the case of size simplification, the proposed approach allows a better preservation the contrast of maxima or minima that have not been totally simplified by the filtering process. In the case of motion simplification, two applications are presented: motion detection for maxima/minima and sequence simplification to improve the coding efficiency of standard encoders.

1. INTRODUCTION

Filtering techniques commonly used in image processing are defined by an input/output relationship that relies on a specific signal called impulse response, window or structuring element. The impulse response of a linear filter defines the filter properties but introduces some blurring in the output image. The major drawback of median filtering is that every region tends to be round after filtering with most commonly used windows (circles, squares, etc.). This effect is due to the shape of the window combined with the median processing. Morphological opening and closing also introduce severe distortions due to the shape of the structuring element. The goal of filter design is to appropriately select the impulse response, the window or the structuring element. However, in the framework of image processing, this selection implies some drawbacks because severe distortions are introduced in the output images.

Connected operators follow a different strategy. These filtering tools do not modify individual pixel values but directly act on the connected components of the space where the image is constant, the so-called *flat zones*. Intuitively, connected operators can remove boundaries between flat zones but cannot add new boundaries nor shift existing contours. The related literature rapidly grows and involves theoretical studies [1, 2, 3, 4, 5], algorithm developments and applications [6, 7, 8, 9, 10].

The goal of this paper is to review the strategy followed by connected operators, in particular those based on reconstruction process, and to present new connected operators for size and motion filtering. The organization of the paper is as follows: Section 2 give a brief overview on connected operators. Connected operators based on anti-extensive reconstruction are discussed in section 3 where new operators for size simplification and motion detection are presented. Section 4 focuses on self-dual reconstruction and levelings. The application of levelings for video pre-processing for compression is discussed.

2. CONNECTED OPERATORS

Gray level connected operators act by merging flat zones. They cannot create new contours nor modify the position of existing boundaries between regions. Therefore, they have very good contour preservation properties. Gray level connected operators originally defined in [1] rely on the notion of flat zones partition. We assume that the connectivity is defined on the digital grid by a translation invariant, reflexive and symmetric relation. Typical examples are the 4- and 8-connectivity. Let us denote by $\mathcal{P}$ a partition and by $\mathcal{P}(n)$ the region that contains pixel n. A partial order among partitions can be created: $\mathcal{P}_1$ "is finer than" $\mathcal{P}_2$ (written as $\mathcal{P}1 \sqsubseteq \mathcal{P}_2$), if $\forall n, \mathcal{P}_1(n) \subseteq \mathcal{P}_2(n)$. The set of flat zones of an image f is a partition of the space, $\mathcal{P}_f$. Based on these notions, connected operators are defined as:

Definition 1 *(Connected operators) A gray level operator ψ is connected if the partition of flat zones of its input f is always finer than the partition of flat zones of its output, that is: $\mathcal{P}_f \sqsubseteq \mathcal{P}_{\psi(f)}, \forall f$*

This definition clearly highlights the region-based processing of the operator since it states that regions of the output partition are created by union of regions of the input partition. An alternative (and equivalent) definition of connected operators was introduced in [4]. This second definition enhances the role of the boundaries between regions and turns out to be very useful to derive the notion of leveling (see Section 4).

Definition 2 *(Connected operators) A gray level operator ψ is connected if $\forall f$ input image and $\forall n, n'$ neighboring pixels:*
$\psi(f)[n] \neq \psi(f)[n'] \Longrightarrow f[n] \neq f[n']$.

This definition simply states that, if two neighboring pixels of the output image have two different gray level values, they have also two different gray level values in the input image, in other words, the operator cannot create new boundaries. There are several ways to construct connected operators. From the practical viewpoint, the most successful strategies rely either on reconstruction processes [6] or on region-tree pruning [8, 9]. In this paper, we focus on the first strategy.

3. ANTI-EXTENSIVE RECONSTRUCTION AND CONNECTED OPERATORS

3.1. The Anti-extensive Reconstruction Process

The most classical way to construct connected operators is to use an anti-extensive reconstruction process. It is defined as follows:

Definition 3 *(Anti-extensive reconstruction) If f and g are two images (respectively called the "reference" and the "marker" image), the anti-extensive reconstruction $\rho^{\downarrow}(g|f)$ of g under f is given by:*

$$\begin{aligned} g_k &= \delta_C(g_{k-1}) \bigwedge f \text{ and} \\ \rho^{\downarrow}(g|f) &= \lim_{k\to\infty} g_k \end{aligned} \tag{1}$$

where $g_0 = g$ and δ_C is a dilation with a flat structuring element defining the connectivity (3x3 square for 8-connectivity or cross for 4-connectivity).

It can be shown that the series, g_k, always converges and the limit always exists. By duality[1], the extensive reconstruction is given by:

Definition 4 *(Extensive reconstruction) If f and g are two images (respectively called the "reference" and the "marker" image), the extensive reconstruction $\rho^{\uparrow}(g|f)$ of g above f is given by:*

$$\begin{aligned} g_k &= \epsilon_C(g_{k-1}) \bigvee f \text{ and} \\ \rho^{\uparrow}(g|f) &= \lim_{k\to\infty} g_k \end{aligned} \tag{2}$$

where $g_0 = g$ and ϵ_C is an erosion with the flat structuring element defining the connectivity.

Note that Eqs. (1) and (2) define theoretically the reconstruction processes but do not provide efficient implementations. Indeed, the number of iterations is generally fairly high. The most efficient reconstruction algorithms rely on the definition of a clever scanning of the image and are implemented by First-in-First-out (FIFO) queues. A review of the most popular reconstruction algorithms can be found in [6].

In practice, useful connected operators are obtained by considering that f is the input image and one has to derive somehow the marker image g. Most of the time, the marker is itself a transformation $\phi(f)$ of the input image f. As a result, most connected operators ψ obtained by reconstruction can be written as: $\psi(f) = \rho^{\downarrow}(\phi(f)|f)$ (anti-extensive operator: $\psi(f) \leq f$), or $\psi(f) = \rho^{\uparrow}(\phi(f)|f)$ (extensive operator: $\psi(f) \geq f$). In the following, we discuss some examples of size and motion simplification.

3.2. Size Filtering

The simplest size-oriented connected operator is obtained by using as marker image, $\phi(f)$, the result of an erosion with a structuring element h_k of size k. It is the opening by reconstruction of erosion:

$$\psi(f) = \rho^{\downarrow}(\epsilon_{h_k}(f)|f) \tag{3}$$

This operator is an opening. By duality, the closing by reconstruction is given by: $\psi^*(f) = \rho^{\uparrow}(\delta_{h_k}(f)|f)$. An example of opening by reconstruction of erosion is shown on the left side of Fig. 1. In this example, the original signal f has 11 maxima. The marker g is created by an erosion with a flat structuring element which eliminates the narrowest maxima. Only 5 maxima are preserved after erosion. Finally, the marker is reconstructed. In the reconstruction, only the 5 maxima that were present after erosion are visible and narrow maxima have been eliminated. Moreover, the transitions of the reconstructed signal correspond precisely to the transitions of the original signal.

[1]that is by changing the sign of the images: $f \leftrightarrow -f$

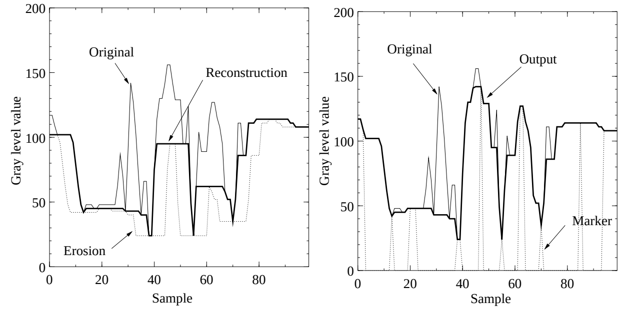

Fig. 1. Size-oriented connected operators: Left) Opening by reconstruction of erosion, Right) New marker indicating where the first reconstruction has not been active and second reconstruction.

As can be seen, the simplification effect, that is the elimination of narrow maxima is almost perfectly done. However, the preservation effect may be criticized: although the contours of maxima are well preserved, their shape and height are distorted. To reduce this distortion, a new connected operator can be built on top of the first one. Let us construct a new marker image, $m[n]$, indicating the pixels where the reconstruction has not been active, that is where the final result is equal to the erosion.

$$\begin{aligned} m[n] &= f[n], \quad \text{if } \rho^{\downarrow}(\epsilon_{h_k}(f)|f)[n] = \epsilon_{h_k}(f)[n] \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{4}$$

This marker image is illustrated on the right of Fig. 1. It is equal to 0 except for the five maxima that are present after erosion and also for the local minima. At that locations, the gray level values of the original image, $f[n]$, are assigned to the marker image. Finally, the second connected operator is created by the reconstruction of the marker, m under f: $\psi(f) = \rho^{\downarrow}(m|f)$.

This operator is also an opening by reconstruction. The final result is shown on the right side of Fig. 1. The five maxima are better preserved than with the first opening by reconstruction. In this example, the first opening by reconstruction is used to define a marker of the interesting maxima and the second reconstruction simplifies the image. The difference between both reconstructions is also clearly visible in the examples of Fig. 2. The first opening by reconstruction removes small bright details of the image: the text in the upper left corner. The fish is a large element and it is not removed. It is indeed visible after the first opening by reconstruction (Fig. 2(center)) but its gray level values are not well preserved. This drawback is avoided by using the second reconstruction. Finally, by duality, closings by reconstruction can be defined. They have the same effect than the openings but on dark components.

3.3. Motion detection of maxima/minima

An interesting application of anti-extensive reconstruction is illustrated in Fig. 3. The goal is to detect the motion of maxima (the dual operator can deal with minima). Assume that we want to detect the maxima following a given motion. The first step consists in computing the temporal gradient in the direction of the motion (in the example of Fig. 3, the motion is static and the maxima to be detected are still). The marker is then created by extracting the meaningful positive components of the gradient. This can be done for example by thresholding. The location of the marked pixels

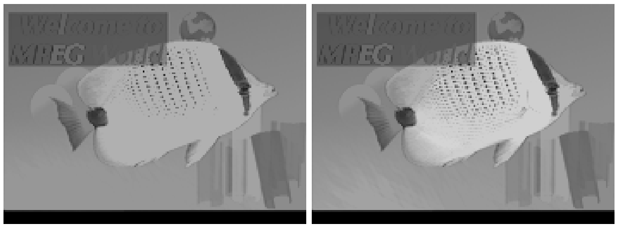

Fig. 2. Size filtering with opening by reconstruction: Left) reconstruction of an erosion (size of structuring element 10x10), and Right) second reconstruction.

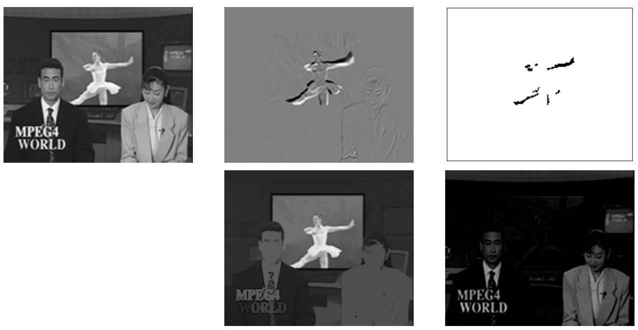

Fig. 3. Motion detection of maxima: Top row: original image, Middle row: temporal gradient, mask of the marker; Bottom row: anti-extensive reconstruction of the marker, difference between the reconstruction and the original images.

are show in black in Fig. 3. The marker image is reconstructed and finally, the maxima of interest are obtained by computing the difference between the reconstructed and original images. As a result the maxima that do not follow the motion are highlighted (see Fig. 3).

4. SELF-DUAL RECONSTRUCTION AND LEVELINGS

4.1. The Self-dual Reconstruction Process

The connected operators discussed in the previous section were either anti-extensive or extensive. In practice, they allow the simplification of either bright or dark image components. For some applications, this behavior is a drawback and one would like to simplify in a symmetrical way all components. From the theoretical viewpoint, this means that the filter has to be self-dual, that is $\psi(f) = -\psi(-f)$. With the aim of constructing self-dual connected operators, the concept of levelings was proposed in [4, 5] by adding some restrictions in Definition 2:

Definition 5 *(Leveling) The operator ψ is a leveling if $\forall n, n'$ neighboring pixels, $\psi(f)[n] > \psi(f)[n'] \implies f[n] \geq \psi(f)[n]$ and $\psi(f)[n'] \geq f[n']$.*

This definition not only states that if a transition exists in the output image, it was already present in the original image (Definition 2) but also that 1) the sense of gray level variation between n and n' has to be preserved and 2) the variation $\|\psi(f)[n] - \psi(f)[n']\|$ is bounded by the original variation $\|f[n] - f[n']\|$. The

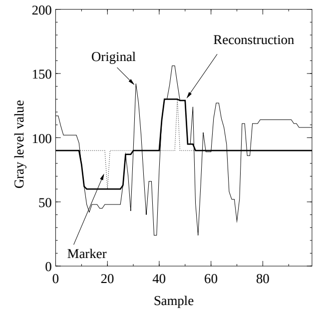

Fig. 4. Example of leveling with self-dual reconstruction.

Fig. 5. Size filtering with leveling: Left) Median filter with a window of size 15x15 and Right) Self-dual reconstruction of the median filter.

most popular technique to create levelings relies on the following self-dual reconstruction process:

Definition 6 *(Self-dual reconstruction) If f and g are two images (respectively called the "reference" and the "marker"image), the self-dual reconstruction $\rho^{\updownarrow}(g|f)$ of g with respect to f is given by:*

$$\begin{aligned} g_k &= \epsilon_C(g_{k-1}) \bigvee [\delta_C(g_{k-1}) \bigwedge f] \\ \rho^{\updownarrow}(g|f) &= \lim_{k\to\infty} g_k \end{aligned} \quad (5)$$

where $g_0 = g$ and δ_C and ϵ_C are respectively a dilation and a erosion with the flat structuring element defining the connectivity (3x3 square or cross).

An example of self-dual reconstruction is shown in Fig. 4. In this example, the marker image is constant everywhere except for two points that mark a maximum and a minimum of the reference image. After reconstruction, the output has only one maximum and one minimum. As can be seen, the self-dual reconstruction is the anti-extensive reconstruction of Eq. 3 for the pixels where $g[n] < f[n]$ and the extensive reconstruction of Eq. 4 for the pixels where $f[n] < g[n]$.

In practice, the self-dual reconstruction is used to restore the contour information after a simplification performed by an operator that is neither extensive nor anti-extensive. Fig. 5 shows an example where the marker image is created by a median filter (which is self-dual operator). This kind of results can be extended to any type of filter and the self-dual reconstruction can be considered as a general tool that restores the contour information after a filtering process. In other words, the reconstruction allows to create a connected version $\rho^{\updownarrow}(\psi(f)|f)$ of any filter: $\psi(f)$.

4.2. Video sequence pre-processing for compression

Levelings are particularly attractive to process difference images. As an application, let us describe a pre-processing strategy that creates video sequences that can be more efficiently compressed by standard coding algorithms such as MPEG-1,2,4 or H.263,+,L. Most of the bits used by the compression algorithm are devoted to the motion compensation error. As a pre-processing technique, we propose to use levelings to simplify the difference between two successive frames of a video sequence, f_t where t denotes the time instant. The difference between two frames is given by Eq. 6 and noted g_t. This temporal gradient can be considered as the compensation error in the case of static areas. It involves noise and error due to the motion of objects, change in lighting conditions, etc. The goal of the pre-processing is to remove from the gradient, g_t, the elements that are not perceptually relevant. To this aim, we extract the relevant minima and maxima of the gradient to create the marker image m_t, see Eq. 7. In our experiment, we have used a simple thresholding technique. The marker is then reconstructed with the self-dual reconstruction process of Eq. 5. This gives the simplified gradient $g_t^{'}$. Finally, the simplified frame $f_t^{'}$ is created by adding the simplified gradient to the previous frame (Eq. 9). Note that in Eqs. 6 and 7, the "previous frame" is not the previous frame of the original sequence, f_{t-1}. It is the previous frame that will be known from the encoder and decoder (because the decoder will also perform a motion compensation), it is therefore the previous pre-processed frame, $f_{t-1}^{'}$.

$$g_t = f_t - f_{t-1}^{'}, \text{ with } f_0^{'} = f_0 \quad (6)$$
$$m_t = \text{relevant maxima and minima of } g_t \quad (7)$$
$$g_t^{'} = \rho^{\updownarrow}(m_t|g_t) \quad (8)$$
$$f_t^{'} = f_{t-1}^{'} + g_t^{'} \quad (9)$$

The interest of this pre-processing is highlighted in Fig. 6 which gives the rate/distortion curves for the original sequence (Threshold $= 0$) and 3 values of the threshold used to define the marker m_t. The values below 10 correspond to sequences that are perceptually equivalent to the original. The original sequence is the *Carphone* sequence in CIF format at 30Hz. The compression algorithm is the standard MPEG-4 codec (simple profile). As can be seen, the pre-processed sequences can be more efficiently compressed in particular for medium to high quality. For this quality range, the reduction is terms of bitrate is between 10 and 20%. In the future, we will investigate how to obtain a similar gain for low bitrate. This will likely involve the inclusion of the motion estimation/compensation process in the simplification strategy.

5. CONCLUSIONS

This paper has discussed connected operators based on reconstruction process. New operators have been proposed to perform size and motion simplification. In the case of size simplification, the proposed approach allows to better preserve the contrast of maxima or minima that have not been totally simplified by the filtering process. In the case of motion simplification, two applications are illustrated: motion detection for maxima/minima and sequence simplification to improve the coding efficiency of classical encoders.

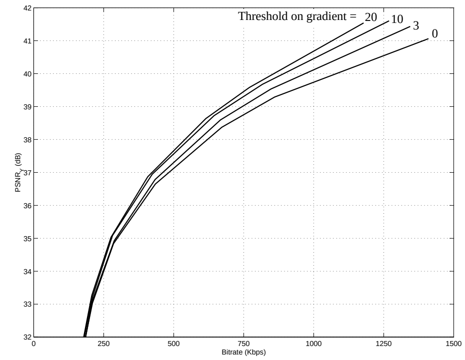

Fig. 6. Rate/Distortion curves for the simplified video sequences.

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