Geometrical Image Filtering with Connected Operators and Image Inpainting

Mariella Dimiccoli [a] and Philippe Salembier [a]

[a]Dept. of Signal Theory and Communications, Technical University of Catalonia
Jordi Girona 1-3, 08034 Barcelona, SPAIN

ABSTRACT

This paper deals with the joint use of connected operators and image inpainting for image filtering. Connected operators filter the image by merging its flat zones while preserving contour information. Image inpainting restores the values of an image for a destroyed or consciously masked subregion of the image domain. In the present paper, it will be shown that image inpainting can be combined with connected operators to perform an efficient geometrical filtering technique. First, connected operators are presented and their drawbacks for certain applications are highlighted. Second, image inpainting methodology is introduced and a structural image inpainting algorithm is described. Finally, a general filtering scheme is proposed to show how the drawbacks of connected operators can be efficiently solved by structural image inpainting.

Keywords: Image inpainting, connected operators, mathematical morphology, partial differential equations

1. INTRODUCTION

Within the framework of Mathematical Morphology, connected operators[1] have proved to be attractive tools for practical image applications due to their fundamental property of simplifying the signal while preserving contour information. This property is a natural consequence of their definition: gray level connected operators filter the image by merging of elementary regions called flat zones. As a result, they cannot create new contours nor shift the position of existing boundaries between regions. However, connected operators may present a drawback in the way they restore the filtered areas. In these areas, the original flat zones are merged into a single one. The gray level value of this new flat zone depends on the gray level values of its surrounding. As a consequence, the filtered areas may not be totally removed and their presence may still be perceptible in the filtered image.

Structural image inpainting has been recently developed within the framework of Partial Differential Equations.[2,3] Inpainting is essentially an extrapolation problem with special focus on situations in which image information is partially missing or inaccessible on certain regions. The term structural refers to a class of algorithms that focuses on restoring geometric structures of the image perceived as lines and object contours, as opposed to texture synthesis algorithms. Many interesting applications of inpainting have been reported, such as image restoration , removal of superimposed text or selected objects, digital zooming and restoration of missing blocks in wireless communication.[3–6] Although a number of techniques exists for the semi-automatic detection of areas to be inpainted, in applications as removal of defects or writings, user interaction is required.

The basic idea presented in this paper is to use image inpainting to perceptually remove areas that have been filtered by connected operators. Equivalently, connected operators are used to automatically define the areas where the inpainting is going to be applied. The organization of the paper is as follows. The next section provides an introduction to connected operators and highlights their drawbacks in terms of perception of filtered areas. Section III is devoted to image inpainting and a practical algorithm is described. In section IV, the proposed filtering scheme is presented and its structure is analyzed. In section V, several filtering examples and applications are reported and a structural image quality assessment[7] is used to benchmark the proposed filtering approach. Finally, section V concludes the paper.

2. CONNECTED OPERATORS

Connected operators have been introduced in image processing as an alternative to classical filtering techniques, which are very closely related to the pixel-based representation of images. The classical processing strategy consists in modifying the values of individual pixels by a signal $h(x)$ defined in a local window. Typical examples include convolution, median filter, and morphological operators based on erosion and dilation, for which the signal $h(x)$ respectively consists of an impulse response, a window and a structuring element. Since the signal $h(x)$ is not related at all with the input signal, its shape introduces severe distortions in the output. Connected operators overcome this drawback by taking a completely different approach: they do not use any specific signal but, in the gray level case, they act by merging flat zones, which are the largest connected components of the space where the image is constant. As a result, no distortion related to a priori selected signal is introduced in the output. This property makes them particularly attractive for image applications requiring high precision on contours as segmentation and analysis.[8–14]

Gray level connected operators rely on the notion of partition of flat zones. Let us denote by P a partition of the space and by $P(x)$ the region that contains the pixel x. Partition P_1 is finer than P_2, if $\forall x P_1(x) \subseteq P_2(x)$. Formally, an operator ψ is connected if the partition of flat zones of its input is always finer than the partition of flat zones of its output. This definition clearly highlights the region-based processing of the operator: regions of the output partition are created by union of regions of the input partition. There are several ways to construct connected operators and many new operators have been recently introduced. From the practical viewpoint, the most successful strategies rely on a reconstruction process or on region-tree pruning.

The most popular technique to define a reconstruction process relies on the so-called *leveling*.[11] If u and v are two images (respectively called the *reference* and the *marker* image), the leveling of v with reference u is defined by:

$$\begin{array}{lcl} v_n & = & \epsilon_0(v_{n-1}) \bigvee [\delta_0(v_{n-1}) \bigwedge u] \\ and & & \\ \rho(v|u) & = & \lim_{n\to\infty} v_n \end{array} \qquad (1)$$

where ϵ_0 and δ_0 represent respectively an erosion and a dilation with square or a cross of 3x3, $\vee$ and $\wedge$ the infimum and supremum and $v_0 = v$. An example of reconstruction is shown in Fig. 1. In this example, the marker image is constant everywhere except for two points that mark a maximum and a minimum of the reference image. After reconstruction, the output has only one maximum and one minimum and their contour coincide with those of the original signals. In practice, useful connected operators are obtained by considering that the marker v is a transformation $\Phi(u)$ of the input image.

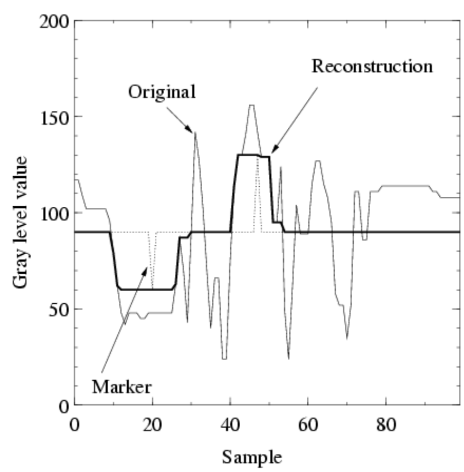


Figure 1. Example of leveling with self-dual reconstruction

The transformation Φ defines the simplification effect and the reconstruction process restores the contour of the flat zones that have not been completely removed by the simplification. As a result, most connected operators obtained by reconstruction can be written as: $\psi(u) = \rho(\Phi(u) \mid u)$. Examples of filtering effect include size oriented (resp. a contrast-oriented) simplification if Φ is an erosion or a dilation with a structuring element (resp. a subtraction of a constant gray level value).

The reconstruction strategy works on a pixel-based representation of images and provides a way to create connected operators. However for certain applications, it lacks of flexibility since the simplification criteria and the resulting operators are in practice limited to be increasing. In the lattice framework, an operator Φ is said to be increasing if it does not modify the order between any pair of elements of the lattice: $\forall u \leq v, \Phi(u) \leq \Phi(v)$. Region-tree pruning strategies offer an alternative way to create connected operators. These strategies are particularly useful in the case of non-increasing criteria. The first step of the approach is to construct a tree representation of the image, which structures the pixels in a suitable way for the filtering process. Then, the simplification is obtained by pruning of the tree. Classical trees to construct connected operators are the Max-tree/Min-tree,[9] the Binary Partition Tree[10] and the Component Tree.[15]

In the case of Max-tree and Min-tree, the tree representation is created recursively by a study of the thresholded version of the image at all possible gray levels. An example of Max-tree is shown in Fig.2. The original

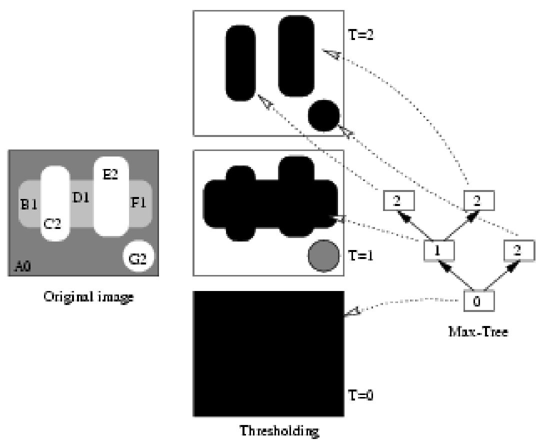


Figure 2. Max-tree representation of images

image is made of 7 flat zones, labeled by a letter $\{A, ..., G\}$. The number following each letter represents the gray level of the flat zone. In our example, the gray level values range from 0 to 2. In the center of the figure the binary images obtained thresholding the image at all possible gray levels are shown. In the right side, the resulting Max-tree representation is presented. The tree nodes represent the set of connected components that arise from thresholding the image at the gray-level indicated by the number inside the square. The tree branches represent the inclusion relationships among connected components following the threshold values. The tree root represents the entire support of the image and the tree leaves represent the regional maxima.

The interest of the tree representation is that the set of possible merging steps is fixed and represented by the tree branches. As a consequence, sophisticated pruning strategies can be designed, allowing to deal in particular with non-increasing criteria. In the increasing case, the increasingness of the criterion guarantees that if a node has to be removed (because the criterion value assessed on this node is below a given threshold) all its descendents have also to be removed. As a result, the pruning strategy is straightforward and consists in merging all nodes where the criterion value is below a given threshold. In the case of non-increasing criteria, there is no relation between the criterion values of a node and of descendents. As a result, the pruning strategy is not straightforward. In practice, the non-increasingness of the criterion implies a lack of robustness of the operator, in the sense that small modifications of the criterion threshold may result in drastic changes in the filtered image. Some practical rules have been reported in the literature to deal with the non-increasing case, ranging from the most straightforward one, which removes a node only if it satisfies the simplification criterion, to the most complex one, which formulates the problem as a dynamic programming issue and efficiently solves it with the Viterbi algorithm.[9,16–18]

Summarizing, the region-tree pruning approach is conceptually more complex than the reconstruction approach. However, it provides more flexibility in the choice of the simplification criterion and also leads to a very efficient implementations of connected operators.

(a) (b)

Figure 3. Original image (a) and image filtered by a size oriented connected operator (b)

Even if connected operators allow image simplification without introducing distortion on the remaining contours, they may not totally remove the presence of simplified areas. Indeed, these areas appear as individual flat zones (because the original flat zones have been merged into a single one) with a gray level value that depends on the surrounding flat zones. Fig.3 (b) illustrates this effect with a size-oriented connected operator: $\psi(u) = \rho(\epsilon(u) \mid u)$ where ϵ is an erosion with a square structuring element of size 5x5. The text "MPEG4 WORLD" on the image has been filtered by merging its flat zones. However, the text is still visible because the letters have been restored with a gray level value that depends of surrounding flat zones. In some applications, we would like the simplification to remove regions from the original image without leaving any perceptual information about them. In the sequel, we study the use of inpainting techniques to estimate the pixel values of areas where flat zones have been merged.

3. IMAGE INPAINTING

3.1. Introduction

In the field of image processing, the term inpainting refers to the task of restoring the pixel values for a missing or consciously masked sub-region of the image domain. Depending on their goals and methodology, available methods can be broadly classified in *structural* or *textural* inpainting. Structural inpainting restores the geometric structures of the image using prior assumptions and boundary conditions. Textural inpainting restores the texture of the image only considering available data from texture exemplars. In some analysis applications, where the goal is to understand the scene, geometric structures such as contours play a key rule. As our goal is to use inpainting to estimate the regions filtered by connected operators, a structural inpainting algorithm will be used.

The basic idea of the algorithms proposed in the literature[2–5,19–23] is to restore the missing regions with the information available from their surrounding. Each image is a 2-D projection of a 3-D scene. As a result, individual objects often have geometric or surface regularity. Generic regularity, that is smooth continuation of geometric contour, is used as basic assumption to define the optimum inpainting. The most impressive results are achieved using the curvature of level sets. In this way the angle with which the lines delimiting the level sets, called level lines, arrive at the occlusion boundary is preserved. The inpainting algorithm used in this paper is a curvature based approach.[4] The authors formulate the inpainting as the problem of restoring, in the regions of missing data, both the geometric and the photometric information represented respectively by the level lines and the gray level values.

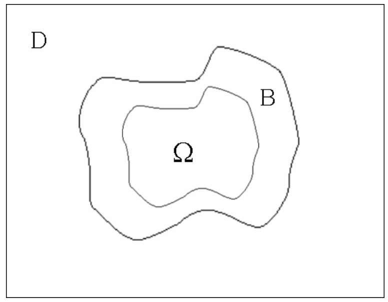


Figure 4. The image domain D with the region to be inpainted Ω and its band B

The image U is modeled as a real function defined in a rectangular subset D of R^2 ($U\colon D \subset R^2 \rightarrow R$). Let Ω be the region to be inpainted, and B a narrow band of pixels surrounding Ω, where the pixel values are known. The algorithm works on $\tilde{\Omega} = (\Omega \cup B) \subset D$. In order to extrapolate the shape information independently from the contrast, the image in $\tilde{\Omega}$ is decomposed into level sets $U_\lambda = \{x \in \tilde{\Omega} \mid U(x) \geq \lambda\}$, which are inpainted individually. Then, the output image in the region of missing data is obtained by stacking the extrapolated level sets. For each level set U_λ the solution on Ω is obtained minimizing the following functional over $\tilde{\Omega}$

$$E_\lambda = \int_{\tilde{\Omega}} |div(\theta)|(a + b|\nabla U_\lambda|)dx + \alpha \int_{\tilde{\Omega}} (|\nabla U_\lambda| - \theta \cdot \nabla U_\lambda)dx \tag{2}$$

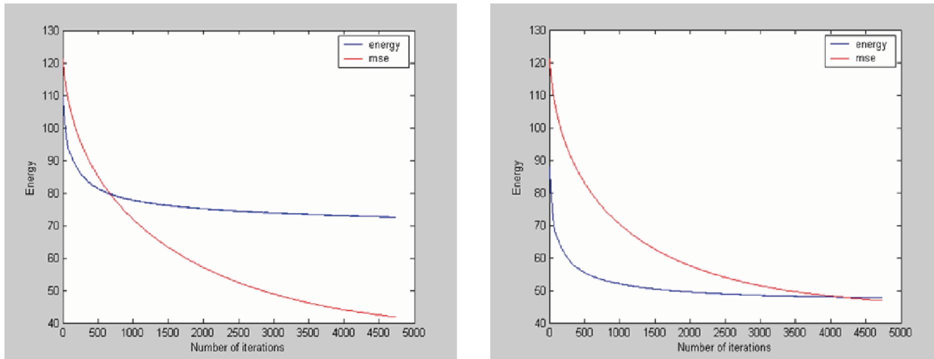
where a, b and α are positive constants and ideally θ represents the normal vector field to the level lines of U_λ, that is

$$\theta = \frac{\nabla U_\lambda}{|\nabla U_\lambda|} \tag{3}$$

In practice, θ is computed by equation (3) only at the initialization time. Once the two variables θ and U_λ have been initialized, they are updated independently from each other and the coupling between them is only indirectly hold by the second integral term of (2), which represents a relaxation of the constraint (3). Intuitively, the geometric quantities which appear in the functional are the curvature of level lines and the perimeter of the discontinuities represented respectively by the quantities $div(\theta)$ and $(a + b|\nabla U_\lambda|)$ in the first integral term. The decision of how to extrapolate the level set is thus based on the total perimeter and total curvature of the discontinuities on $\tilde{\Omega}$.

3.2. Optimization algorithm

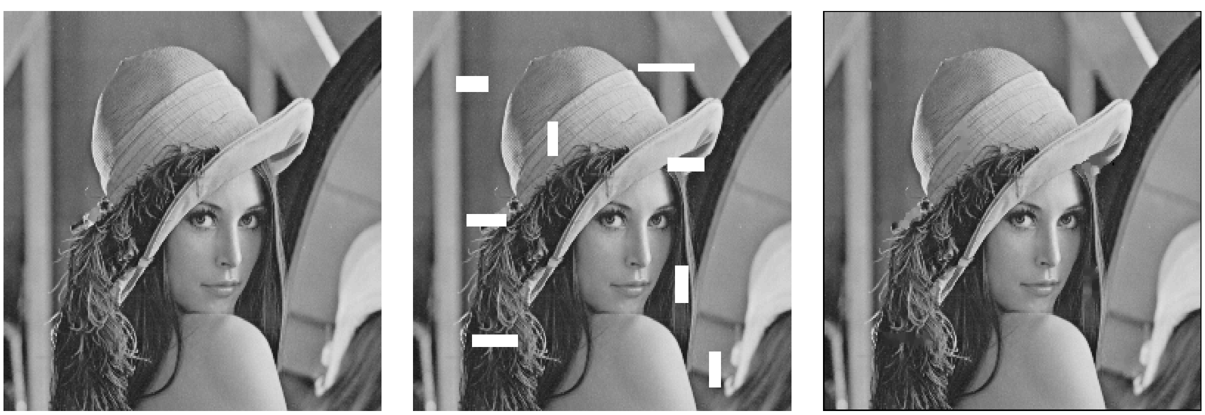
Numerically, the minimization of the functional (2) is computed solving a variational problem via its gradient descent flow, which leads to a set of coupled second order partial differential equations: one for the gradient orientations and one for the gray levels. The resulting numerical algorithm is iterative and the iteration is realized by the introduction of a time variable that is discretized using an implicit stepping scheme.[4] Each iteration of the algorithm involves two broad steps. First, the functional is minimized with respect to θ, allowing to extend the normal field into the region of missing data. Second, with the new values of θ, the functional is minimized with respect to U_λ, allowing to extend the gray level values of the image in the direction pointed by the new values of θ. To run the algorithm, the pixel values in Ω are initialized with the value 0,5. Once the convergence has been reached the image is binarized setting to 1 all pixels in Ω with value greater than 0,5 and to 0 the remaining pixels. Considering the variation of the functional as a function of the iteration number, the stopping criterion is defined by a threshold on its slope. In practice, when the local slope of the functional variation is sufficiently close to 0, it is assumed that the algorithm has converged.



(a) (b)

Figure 5. Energy and mean square error for two different level set

Fig. 5 shows two graphs representing the energy of the functional and the mean square error between the original and the partially inpainted level set as a function of the number of iterations. The two graphs ((a) and (b)), correspond to two different level sets of the same image. In this experiment, the ideal inpainted image, called original image, was known (Fig. 6(a)) and masked (Fig. 6(b)). As a result, the mean square error between the original and inpainted images defines how close the solution is from the ideal one. As can be observed in both graphs, a decrease of the energy corresponds to a decrease of the mean square error. This motivates the proposal of using the slope of the energy function as termination criterion. In addition, although the two level sets have the same size, the evolution of the energy function and thus the convergence velocity is different for the two level sets. The reason is that, the number of iterations depends not only on the size of $\tilde{\Omega}$, but on the complexity of the level lines in the band.



(a) (b) (c)

Figure 6. Original image (a), image masked (b), image inpainted (c)

In Fig. 6(c), an example of result obtained using the inpainting algorithm is shown. As can be observed, the approach gives results of high quality for relative small regions.

4. PROPOSED APPROACH

In this section, a new filtering strategy involving connected operator and inpainting is proposed. The approach is general and can be applied to any connected operator.

The proposed filtering scheme (Fig. 7) involves two major steps: simplification by connected operators and restitution of the perceptually most important simplified regions by inpainting. As stated above, structural inpainting algorithms extend the geometric structures from the band to the domain to inpaint. The presence of noise or small details in the band (smaller or equal to a structuring element of size 3x3) may affect the effectiveness of the inpainting by disturbing the capture of the geometry.

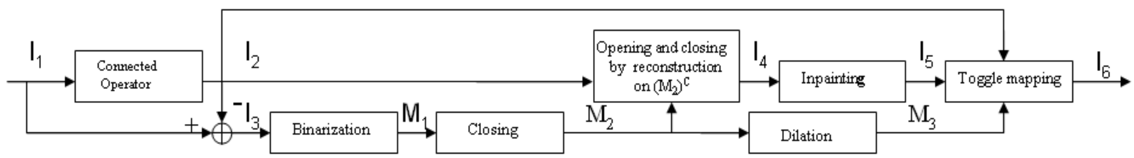
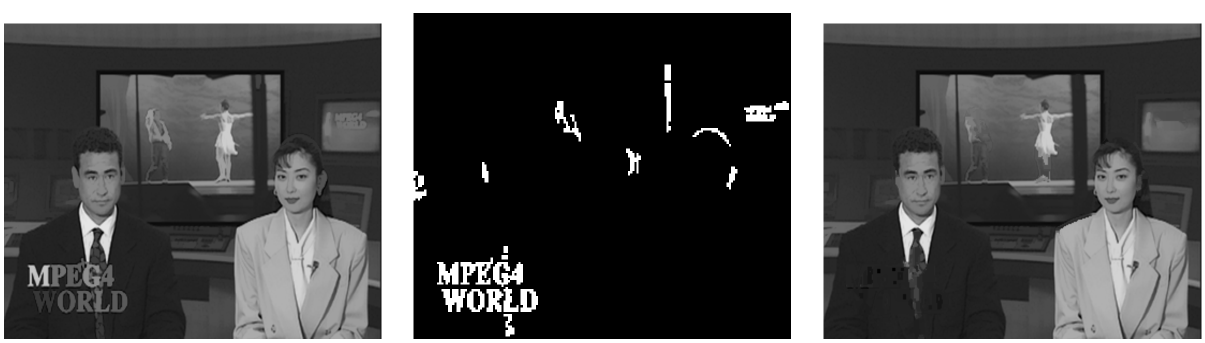


Figure 7. Proposed filtering scheme. I refers to an image.; M refers to a mask; M^C refers to the complement of M

In order to extend in the regions to be inpainted only the most meaningful gray levels and contours, we apply an opening by reconstruction of erosion (structuring element of size 3x3), followed by its dual, the closing by reconstruction dilation. After inpainting, both the smoothed version of the band and the inpainted content of Ω are inserted in the output image. This strategy guarantees that inpainted area will be perceived as a smooth extension of the visual information contained in the band.



(a) (b) (c)

Figure 8. Image after simplification by the connected operator (a), inpainting mask (b), filtered image using the proposed approach (c)

Fig. 8 shows an example of results obtained using the proposed scheme. First, the original image (see Fig. 3 (a)) is filtered using a size-oriented connected operator: an opening by reconstruction of erosion with a structuring element of 5x5 (Fig. 8 (a)). Second, a mask, defining the perceptually most important regions that have been removed by the connected operator, is computed and the regions that are very close to each other are merged by a closing. The resulting mask is shown in Fig. 8 (b). Third, the image is smoothed in the areas surrounding the mask using an opening by reconstruction followed by its dual, the closing by reconstruction. Fourth, the regions of this smoothed image marked by the mask are inpainted. Finally, a toggle mapping is performed to copy the regions estimated by inpainting, as well as their smooth bands in the output image. As can be noticed, the connected components marked by the mask shown in Fig. 8 (b), as for instance the writing and the legs of the dancer, visible in Fig. 8 (a), are no longer perceived in Fig. 8 (c), since they have been completely removed by inpainting.

5. EXPERIMENTAL RESULTS

5.1. Examples with various connected operators

This section presents examples of results using the proposed filtering approach. A first example has been shown in the section 4. The goal was to remove small and bright objects from the image. As a consequence a size oriented connected operator such as the opening by reconstruction of erosion, has been used. The next example involves a contrast-oriented connected operator $\Phi(u) = \rho(u - \lambda \mid u)$ where λ is a constant value.

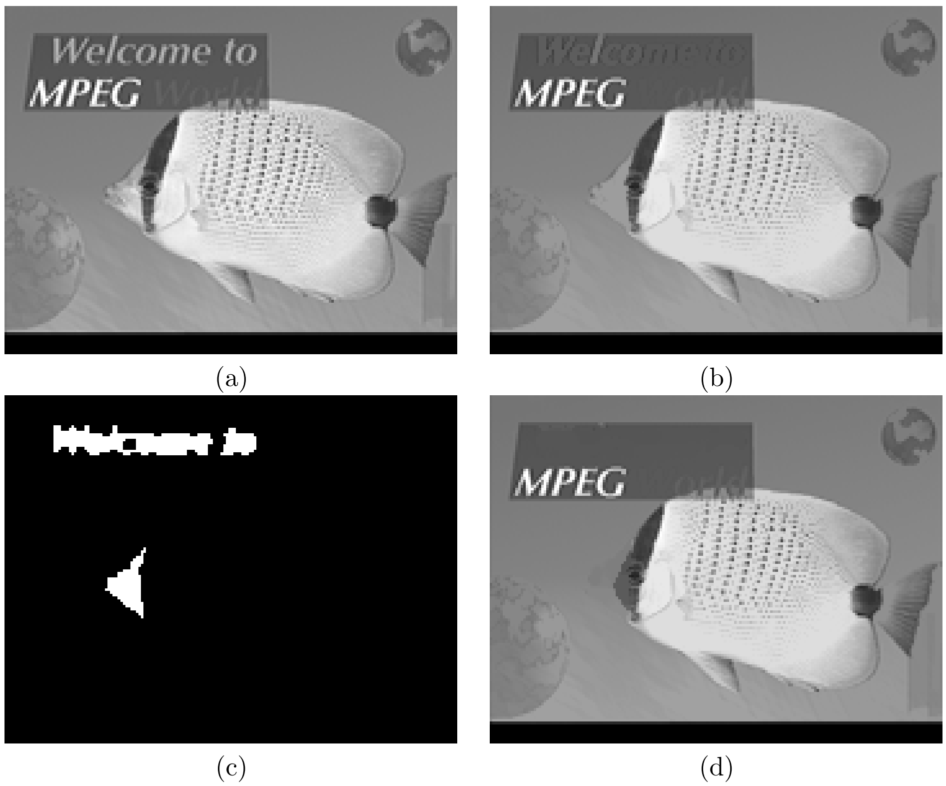


Figure 9. Example of filtering involving a contrast-oriented connected operator: (a) original image; (b) contrast oriented conneted operator; (c) inpainting mask; (d) proposed filter

The goal of this operator is to remove connected component with contrast inferior or equal to λ. The filtered image Fig. 9(b) has been obtained using a contrast-oriented connected operator with $\lambda = 100$. As can be observed, the writing "Welcome to" and part of the of muzzle of the fish have been filtered but their presence is still perceptible in the filtered image. To completly remove their perceptual presence, inpainting is applied to the regions marked by the mask Fig. 9(c). As can be observed in Fig. 9(d), the proposed filtering scheme has efficiently removed the regions marked by the mask.

In the next example, the goal is to remove the dark sticks from the image shown in Fig. 10 (a). To this goal, a size-oriented connected operator ($\psi(u) = \rho(\delta(u) \mid u)$) (closing by reconstruction of dilation by a structuring element of size 9x9) has been used. As can be observed in Fig. 10 (b), although most dark sticks have been filtered, their presence in the filtered image is still strongly perceptible. Whereas, when applying inpainting to the regions marked by the mask (Fig. 10 (c)), the i dark sticks are successfully removed (Fig. 10 (d)).

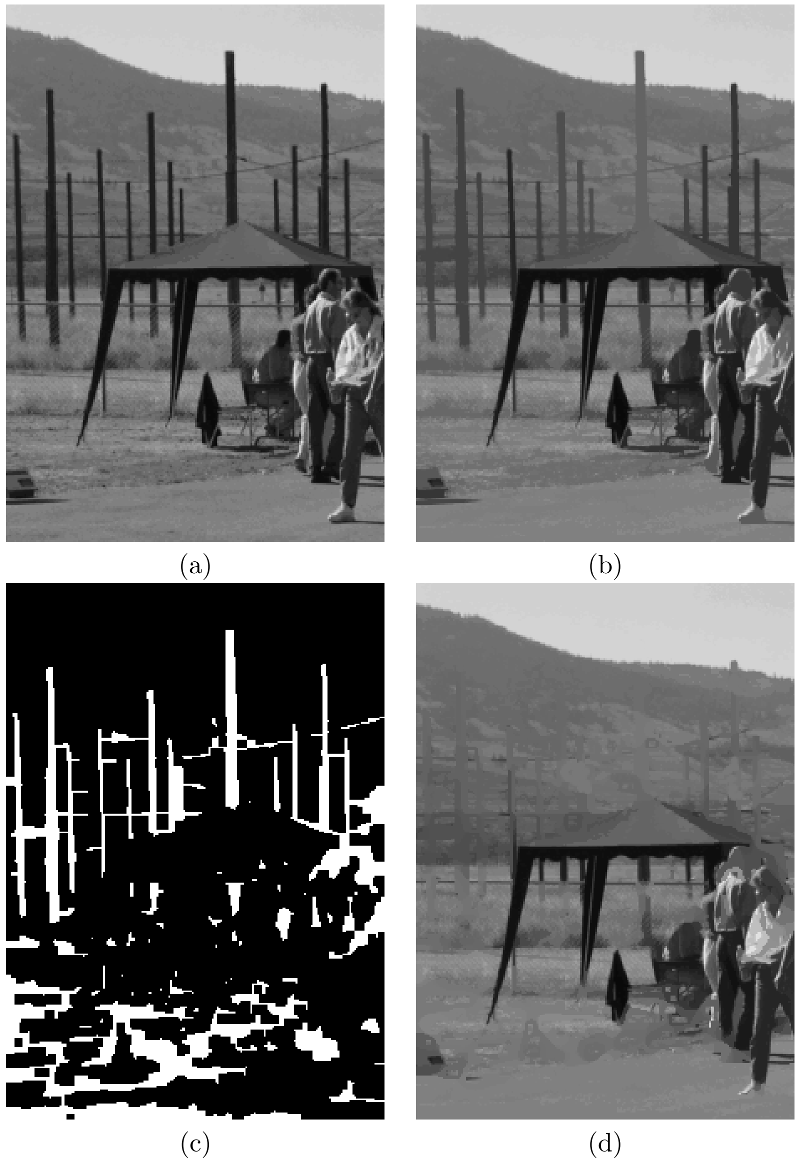


Figure 10. Example of filtering involving a dual size-oriented connected operator: (a) original image; (b) dual size-oriented connected operator; (c) inpainting mask; (d) proposed filter

In next example, the objective is to remove the superimposed lines in the image shown in Fig. 12 (a). Due to their shape, the white lines are suited to be filtered by a complexity oriented connected operator.[24] The complexity criterion is based on a measure of the ratio between the perimeter P and the area A of the connected component. Intuitively, if a connected component has a small area but a very long perimeter, it correspond to a complex object. However, since the image is highly textured, a complexity criterion alone would extract also most of the texture.

In order to extract the white lines, preserving the texture information, a more complex criterion has to be used (see Fig. 11). First, the image shown in Fig. 12 (a) is filtered using a contrast-oriented connected operator, with $\lambda = 170$. In this way, most of texture is extracted and the white lines are preserved. Second, the resulting image is filtered by a complexity criterion, wich extracts the connected components with complexity superior or equal to 42. As a result, the white lines as well as the complex connected components corresponding to texture

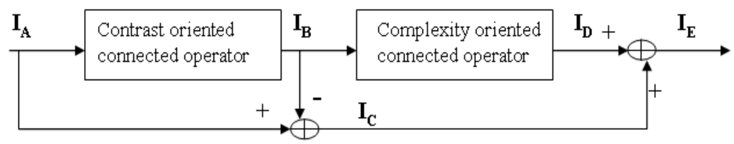
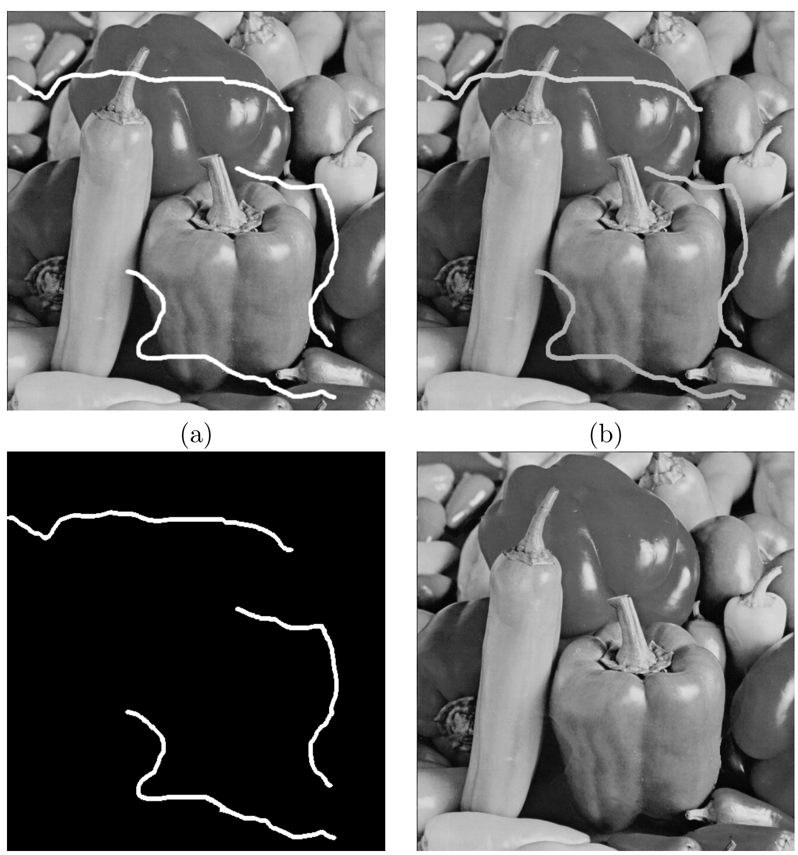


Figure 11. Connected operator used to extract the white lines

more constrasted than $\lambda = 170$ are extracted. Note that the complexity criterion is not increasing because there is no a priori relationship of complexity between two connected components R_1, R_2 such that $R_1 \subset R_2$. In order to deal with this nonincreasing criterion a pruning strategy on a Max-tree has been used. Furthermore, to preserve the highly contrasted characteristic of the superimposed lines, the "Max" decision has been employed. The "Max" decision is defined as follows: a node C for which the evaluation of the criterion is lower than a given threshold is not removed if at least one of its descendents has to be preserved. Finally, the difference between the original image and the image filtered by the contrast-oriented connected operator is added to the image filtered by the complexity criterion to restore the texture.



(a) (b) (c) (d)

Figure 12. Example of filtering involving the connected operator in Fig. 11:(a) original image;(b) connected operator described in Fig. 11; (c) inpainting mask; (d) proposed filter

As can be observed in Fig. 12 (b) the white lines have been filtered and most of the texture preserved. However, the perceptual presence of the superimposed lines is still strongly perceptible. Applying inpainting only to the perceptually most important filtered region (Fig.. 12 (c)) a much better result is obtained (Fig.. 12 (c)).

5.2. Filtering quality assessment using SSIM index

The goal of this subsection is to evaluate the effectiveness of the proposed filtering approach using a visual quality assessment such as the SSIM (Structural Similarity Image Measure).[7,25] The SSIM has been recently introduced in image processing to automatically predict the perceived image quality and it is becoming very popular to optimize and benchmark image processing algorithms. Intuitively, the SSIM considers image degradations as perceived changes in structural information variation. In fact, since the human vision is highly sensitive to structural information, a measure of the structural information change should provide a good approximation to perceived image distortion. In practice, the SSIM quantify the differences between a distorted image and a reference image independently from average luminance and contrast. The SSIM is a full-reference image quality assessment and, in filtering applications, the reference image is generally not available. To overcome this problem the effectiveness of the proposed filtering approach has been tested for the last example described in section 5, for which the original version of the image (image without the superimposed lines is available). In Table 1, the value of the SSSIM obtained using different filtering strategies are reported. The dynamic range of the SSIM is from 0, to 1 as the similarity increases and becomes equal to 1 for two identical images. As can be observed the proposed scheme drastically reduces the SSIM, meaning that the results obtained using the approach proposed in this paper are consistent with the qualitative visual appearance.

Table 1. This table shows the SSIM values obtained using different filtering approaches.

Type of filter	**SSIM value**
Low pass filter (5x5 average)	0.0646
Median filter (5x5)	0.8744
Connected Operator	0.9157
Proposed Approach	0.9917

6. CONCLUSIONS

In this paper, the usefulness of combining connected operators and image inpainting for image filtering has been presented and discussed. The proposed filtering scheme involves two broad steps: first, the image is simplified using connected operators. Second, the perceptually most important filtered regions are estimated using inpainting. The mask marking the regions to be inpainted is automatically computed and no user interaction is required. Comparative experiments have shown that the proposed scheme outperforms early filtering strategies in term of structural perceptual quality. The presented approach is general in the sense that any connected operator with any simplification criterion can be used. As a result, it is suitable for a large set of advanced filtering applications such as objects, writing or defects removal. The extension of the morphological framework through inpainting methodology for image filtering seems to be a very interesting field of research.

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